Preface

Marine Cybernetics is a multidisciplinary education and research program offered by the Department of Engineering Cybernetics and the Department of Marine Technology, the Norwegian University of Science and Technology (NTNU).

The material presented is intended for use as Lecture notes in the course entitled TMR4240 Marine Control Systems and partly in TMR4243 Marine Control Systems II. The courses are given on graduate level for MSc degree for specialization in Marine Cybernetics. It is believed that these courses in addition to the course TTK4190 Guidance and Control will give the students good insight into the design of marine control systems. Besides several dedicated courses on dynamics, hydrodynamics, machinery systems, nonlinear control, stochastic control, instrumentation systems and computer science are given.

It is assumed that the students already have acquired themselves basic background in automatic control theory and mathematical modelling of mechanical systems.

This is the third version of the lecture notes and contains minor updates from the previous one issued in 2011.

The students are greatly acknowledged for comments providing continuously improvements.

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Chapter 1

Introduction

1.1 Marine Cybernetics

Marine control systems or Marine cybernetics is defined to be the science about techniques and methods for analysis, monitoring and control of marine systems.

The main application fields for marine control systems are the three big marine industries: Sea transportation (shipping), offshore oil and gas exploration and exploitation and fisheries and aquaculture. So far most of the examples are collected from the mature industries like shipping and offshore oil and gas exploration and exploitation. It is believed that these industries will be even more technology demanding with focus on safe and cost effective solutions in the years to come. More of the offshore activities is assumed to take place in deeper water based on floating solutions in combination with subsea installations. The need for conducting all-year marine operations in the ocean space will increase and motivate to the development of underwater robotics with increased autonomy. Due to increased industrialization within the fisheries and aquaculture, the industrial content and thereby the introduction of advanced technology are expected to increase. It is foreseen that there is a huge potential for technology transfer among all the marine industries.

Concerning marine control systems it is suggested to divide the control structure into two main areas: real-time control and monitoring and operational and business enterprise management, see Figure 1.1. We will here in particular focus on the various aspects related to the design of real-time control and systems. The integration of real-time systems with operational management and business transactional systems is by the automation industry denoted as Industrial IT.

The real-time control structure is as shown in Figure 1.1 divided into low-level actuator control, high-level plant control and local optimization. We will in the text use demonstrating examples from the offshore oil and gas industry. In particular, examples with dynamically positioned (DP) offshore vessels will be used. A DP vessel maintains its position (fixed location or pre-determined track) exclusively by means of active thrusters. Position keeping means maintaining a desired position in the horizontal-plane within the normal excursions from the desired position and heading. The real-time control structure for a DP system may then consist of (Sørensen [299] and [300]):

- **Actuator control**: The actuators for DP systems are normally thrusters, propellers, and rudders. Local control of propellers and thrusters may be done by controlling e.g. the speed
(rpm), pitch, torque, and power or combinations of these. Dependent on the actuators are mechanically, hydraulically and/or electrically driven controllers with different properties will be used.

- **Plant control**: In station keeping operations the DP system is supposed to counteract the disturbances like wave (mean and slowly varying), wind and currents loads acting on the vessel. The plant controller calculates the commanded surge and sway forces and yaw moment needed to compensate the disturbances. A multivariable output controller often of PID type using linear observers e.g. Kalman filter or nonlinear passive observers may be used. Setpoints to the thrusters are provided by the thruster allocation scheme.

- **Local optimization**: Depending of the actual marine operation the DP vessel is involved in optimization of desired setpoint in conjunction with appropriate reference models for e.g. drilling operations, weather vaning, pipe laying, tracking operations, etc. are used. Notice that in the guidance, navigation and control literature local optimization corresponds to the guidance block.

As the controllers rely on proper measurements to work on, *signal processing* is of vital importance for the stability and robustness of the control system. This will be the first topic of the text.

### 1.2 Main Topics

The topics covered here are:
• Chapter 2 chapter gives an overview of marine control systems exemplified by DP vessels with electrical propulsion systems. Design aspects related to high-level vessel control such as power management and DP are shown. Rules and regulations including testing and verification of marine control systems based on Hardware-In-the-Loop (HIL) testing will also be introduced.

• Chapter 3 gives an introduction to electrical installations on marine vessels. Electric installations are present in any ship from powering of communication and navigation equipment, alarm and monitoring system, running of motors for pumps, fans or winches, to high power installation for electric propulsion. Successful design of control systems for such vessels rely on understanding of the electrical equipment and components used and the electrical systems for power generation, distribution and power consumption. Furthermore, successful solutions for vessels with electric propulsion are found in environments where naval architects, hydrodynamic and propulsion engineers, and electrical engineering expertise cooperates under constructional, operational, and economical considerations.

• Chapter 4 introduces the reader to the basics in linear system theory and computer-controlled systems. Today most of the control systems are implemented in computers run by digital processors. Computers will operate on continuous-time processes at discrete-time sampling instants. Therefore, computer-controlled systems can be regarded as discrete control systems. Discrete systems could in many cases be viewed as approximations of analog systems. Computer-controlled systems may introduce deteriorations of the control objective, if not accounting for the fact that they operate on discrete instants of processes which are time-continuous by nature.

• Chapter 5 is about signal quality control and fault detection. All industrial control systems contains several functions for signal quality checking. The performance of the control system depends on these functions operate properly. In this chapter the most important signal processing properties for the purpose of control will be treated.

• In Chapter 6 the most common analog and digital filters in addition to observer theory are presented. The Kalman filter is also derived. Kalman filters have successfully been used in marine control systems since the late 1970s.

• In Chapter 7 the mathematical models used in the design of dynamic positioning systems and thruster assisted position mooring systems are formulated. To readers unfamiliar with hydrodynamics, the models may look difficult. However, the intention here is to focus on the structural properties of the models important for control, rather than going into details about calculation of the hydrodynamic coefficients. If the reader is inspired to enter this field any further, the author is convinced that this will improve the control system designs even more.

• Chapter 8 contains a comprehensive introduction to dynamic positioning (DP) and thruster assisted position mooring (PM) including design of observers and controllers. In addition an introduction to supervisory-switched control is given for multi-objective control and expanding the operational window handling extreme seas

• Chapter 9 on propulsion and thruster controls show the importances of considering low-level control designs of the actuators ensuring that the high-level control demands are fulfilled according to the control objective.
• Chapter 10 presents a new field of research concerning marine control systems applied on ocean structures. It is expected that control of so-called flexible systems will increase in the future due to increased water depth for offshore operations, new materials and applications.

• Chapter 11 is about motion control of high speed craft. In particular the chapter presents how flexibility in the air cushion pressure dynamics will have impact on the design of a ride control system for motion damping of a high speed craft called Surface Effect Ships (SES).
Chapter 2

Marine Control Systems

2.1 Introduction

The history of automated closed-loop ship control started with Elmer Sperry (1860-1930), who constructed the first automatic ship steering mechanism in 1911 for course keeping (Allensworth [5] and Bennet [22]). This device is referred to as the ”Metal Mike”, and it was capturing much of the behavior of a skilled pilot or a helmsman. ”Metal Mike” did compensate for varying sea states by using feedback control and automatic gain adjustments. Later in 1922, Nicholas Minorsky (1885-1970), presented a detailed analysis of a position feedback control system where he formulated a three-term control law which today is refereed to as Proportional-Integral-Derivative (PID) control (Minorsky [190]). These three different behaviors were motivated by observing the way in which a helmsman steered a ship.

This chapter gives an overview of marine control systems exemplified by dynamically positioned (DP) vessels with electrical propulsion systems. Design aspects related to high-level vessel control such as power management and DP are shown. Design requirements and ability for reconfiguration accounting for physical segregation and redundancy of power, propulsion and automation systems will be presented. Rules and regulations including testing and verification of marine control systems based on Hardware-In-the-Loop (HIL) testing will also be introduced. Other marine control systems; propulsion control systems, control of slender ocean structures, motion control systems for high speed crafts and details on DP systems will be presented in separate chapters later in the text. The design steps combining software (SW) and hardware (HW) architecture, modeling, sensor processing, controller design, simulation and testing will more or less follow the same principles. However, requirements to performance and redundancy, and thereby complexity, may differ for the various marine control system applications.

Offshore exploration and exploitation of hydrocarbons have opened up an era of DP vessels. Currently, there are more than 2000 DP vessels of various kind operating worldwide. DP systems are used for a wide range of vessel types and marine operations:

- **Offshore oil and gas industry**: Typical applications in the offshore market are offshore service vessels, drilling rigs and drilling ships, shuttle tankers, cable and pipe layers, floating production off-loading and storage units (FPSOs), crane and heavy lift vessels, geological survey vessels and multi-purpose vessels. Cable and pipe laying are typical operations which also need tracking functionality.

- **Shipping**: Currently there is a trend towards more automatic control of marine/merchant
vessels, beyond the conventional autopilot. This involves guidance systems coupled to automatic tracking control systems, either at high or low speed. In addition, more sophisticated weather routing and weather planning systems are expected. Automatic docking systems, and a need for precise positioning using DP systems when operating in confined waters will become more used.

- **Cruise ships and yachts:** The cruise and yacht market also make use of more automatic positioning control. In areas where anchors are not allowed due to vulnerable coral reefs, DP systems are used for station keeping. Precise positioning is also required for operating in harbors and confined waters.

- **Fisheries:** Application of more sophisticated guidance, navigation and control systems for ships during fishing are motivated by the need for precise positioning, reduced fuel consumption and intelligent selective fishing.

Electric propulsion is not a very new concept. It has been used as early as in the late 19th century. However, only in few vessels until the 1920s where the electric shaft line concept enabled the design of the largest Trans-Atlantic passenger liners. Variable speed propulsion was used in some few applications during the 1950s and 1960s, while, first when the semiconductor technology became available in large scale commercial applications, this technology became acceptable for a wide range of applications. The introduction of AC drives and podded propulsors was another shift in technology that led to a rapid increase in the use of electric propulsion through the last 15-20 years. Typically, ships with electric propulsion tend to have more system functionality implemented in integrated automation systems, partially because such functionalities are regarded to be necessary for safe and optimal operation, but also because the electric propulsion plant enables the use of such functions. In the commercial market the offshore vessels in addition to cruise ships and ice breakers have been technology drivers concerning automation, power and propulsion systems. They are characterized by the required ability to conduct complex marine operations, operational availability, safety focus, cost effectiveness and flexibility in operational profile concerning transit, station keeping, maneuverability and to some extent also a significant vessel or process load system. These rather complex power plants opened up for an increasing use of fully all-electric ships and the introduction of fully integrated computer-controlled systems in order to operate safely and cost efficiently. Such concepts are today applied in an increasing number of ship applications.

Consequently, the complexity has also increased with a variety of solutions consisting of stand-alone systems, partly integrated systems to fully physical and functional integrated systems. Up to now integrated automation systems have been proprietary with a limited number of vendors. However, in the automation industry it is a trend towards openness in communication protocols and network. How this will influence on the technology solutions and responsibilities for multi-vendor integration systems is still a subject for discussion. During the late 1990s the introduction of low-cost off-shelf computers not originally designed for automation purposes have also been taken more into use. This development is driven by the need to make more cost efficient solutions. Nevertheless it also creates new issues of concern:

- New procedures for design and specification that considers compatibility and integration aspects.

- Failure analysis and test methods, adequate to ensure fault-tolerance in the overall system.
2.2 System Overview

A DP offshore vessel, in this case a drilling rig as illustrated in Figure 2.1, may comprise the following sub-systems:

- **Power system.**
- **Marine control or automation system.**
- **Positioning systems** comprise **dynamic positioning (DP) system** for automatic station keeping and low speed manoeuvring of free floating vessels or **thruster assisted position mooring (PM) system** for anchored vessels. Sensors and position reference systems. Hardware, software and sensors to supply information and/or corrections necessary to give e.g. accurate vessel motion, position and heading measurements.
- **Automatic sailing system** and **nautical system** including autopilot, radar, automatic identification system (AIS), and GPS for automatic course keeping and course changing for transit operations.
- **Joystick system** for manual control of the thruster system for positioning of the vessel in surge, sway and yaw/heading. Often automatic heading control function is included. One should notice that the operator does not control each thruster individually. The operator command a desired thrust in surge, sway and yaw. Then the thruster allocation algorithm map the desired thrust in surge, sway and yaw to individual thrust set-points of the enabled thruster.
- **Manual thruster control system** includes individual control of each thrusters.
- **Propulsion and thruster system** involves all components and systems necessary to supply a vessel with thrust for dynamic positioning, maneuvering and transit. For details the reader is referred to Chapter 9.
- Equipment packages and systems for e.g. drilling, off-loading, crane operations, oil and gas production.
- Mooring system (applicable for moored vessels only). Generally, a mooring system consists of \( n \) lines connected to the structure and horizontally spread out in a certain pattern. At the seabed the lines are connected to anchors. **Spread mooring** systems are used both for semi-submersibles and turret-anchored ships. In the latter case the anchor lines are connected to a **turret** on the ship, which can be rotated relative to the ship. The number of anchor lines may vary, typically from 6 to 12. The length of the anchor lines are adjusted by winches and determines the pre-tension and thus the stiffness of the mooring system.
- Safety systems.
- Auxiliary systems, such at heating, ventilation and air condition (HVAC), water cooling, hydraulic systems, etc.
2.3 Power System

Power system comprises all units necessary to supply the vessel with power. The power generation and distribution systems are divided into the following main parts, for details see Chapter 3. The power generation and distribution systems are divided into the following main parts (Hansen [105]):

- A power generation plant with prime mover and generators. In commercial vessels, the prime movers are typically medium speed diesel engines, due to high efficiency and cost efficiency. Gas turbines, and also steam turbines, are used in some ship applications due to their advantages in weight and size. There are intensive research and development in the field of LNG and feasible fuel cells, which in some future also can be applied in a wider scale than of today. In addition, combined power plants are also looked more into.

- An electric power distribution system with main and distribution switchboards. The distribution system is typically split in two, three or four sections, with the possibility to connect and split the sections by use of bus ties or bus feeders. With a proper protection system and operational philosophy, such systems may have the flexibility of operating in a common, connected mode for optimizing the energy production in normal modes, while
isolating faulty parts of the system and obtain the intended redundancy without loss of maneuverability or station keeping ability by automatic splitting and segregation of the system.

- Transformers for feeding of alternate voltage levels in main or low voltage switchboards and motor control centers.
- Uninterruptible power supply of sensitive equipment and automation systems.
- Rotating converters for frequency conversion and clean power supply.
- Cabling, including cable routing and segregation.

In many ships there is a vessel or process plant system, which is critically dependent on the electrical power system. Such systems can be:

- Hotel loads, e.g. for cruise and passenger vessels.
- Crane and winch systems, e.g. for support vessels.
- Drilling systems, e.g. for drilling rigs.
- Oil and gas production, for FPSOs.
- Load/cargo handling systems, for cargo vessels and tankers.

Even though these systems may not be directly critical for the safe maneuverability of the vessel, there may be serious consequences by loss of controllability, both with concern to safety, environmental or economical consequences. Hence, redundancy and segregation in the process and automation systems may be equally important as in the electrical power and propulsion system. This can be explained by the redundant configuration of a drilling drive system, with dual independent power feeding and possibility to segregate into independent sections either in normal or faulty conditions, as shown in Figure 2.2

2.4 Propulsion System

The propulsion system consists typically of prime movers as diesel engines, generators, transmissions and thrusters. A thruster is here defined as the general expression for a propeller unit. A ship can be equipped with several types of thrusters. Conventional ships typically have a main propulsion unit located aft. Traditionally, shaft line propulsion with controllable pitch or fixed pitch propellers is applied, with rudders to direct the thrust. Another common type is the tunnel thruster which is a propeller inside a tunnel that goes through the hull and produces a fixed-direction transverse force. A third type is the azimuth (rotatable) thruster, which can produce thrust in any direction. While the main and the tunnel thrusters have fixed force directions, the direction of the azimuth thruster can be changed, either manually by the operator or automatically by the positioning control system. Rudders in combination with main propellers can also be used actively in the positioning system to produce a transverse force acting on the stern. In positioning systems the main objective is to control the thruster's force. However, since this resulting force cannot be measured directly, it is common to control the thruster revolution
speed (RPM control), the pitching of the propeller blades (pitch control), or by these in the combination (consolidated control). The servo mechanisms for the propulsion devices must be designed to give accurate and fast response. This is often referred to as low-level thrust control, which will be treated in more detail in the Section 9.

2.5 Marine Automation System

2.5.1 Overview

In marine industry, there are two main directions of solutions for automation systems:

- Integrated automation systems.
- Stand-alone automation systems.

The two different classes of solutions are found in most kind of vessels. However, since complex functionalities and tasks are handled better by the integrated solutions, this is more often found in the vessels with complex and advanced propulsion and station keeping installations. In the following integrated automation system is treated.

Marine automation or vessel control system comprises:

- Control functions for e.g. HVAC control, cargo and ballast control, emergency shutdown and fire and gas detection, off-loading control, engine control, etc.
• Power management system (PMS) for handling of generators, black-out prevention, power limitation, load sharing and load shedding. For advanced vessels more sophisticated energy management systems may be used for intelligent power planning and allocation.

• The operator’s user interface to the automation system is through the Human-Machine Interface (HMI) with display systems and operator panels, often denoted as operator stations.

• Centralized computers with scalable CPU processing and I/O capacities, often denoted as controllers or process control stations (Figure 2.3).

• Distributed computers or PLCs, typically with local control and interfaces to process and to centralized computers.

• Communication buses at the different levels of control.

• Associated cabling, segregation and cable routing.

With today’s standardization on communication in automation systems, it is relatively easy to connect various sub systems into a network communication system. The evolution of interconnecting systems in such networks has, today, developed to be an international standard of many new buildings. It is a challenge of designers, to distinguish between the essential and important communication, and to define the right level of redundancy and inter-dependency of sub-systems.
2.5.2 Data Network and Process Stations

On larger systems the power, automation and the various parts of the positioning system are connected by data networks, and the positioning control systems are implemented in the local process stations. It is important that these systems meet the necessary redundancy level and communication real-time requirements, such that the positioning system is reliable. Today it has been common to integrate all automation system into a common plant network (Figure 2.3).

2.5.3 Operator Stations and HMI

The operator’s link to the positioning system is through the Human-Machine Interface (HMI) on the operator stations. On the operating stations, it is common to arrange a joystick system, giving the operator the opportunity to direct control the thruster forces in surge and sway and moment in yaw. The HMI should be user-friendly and sufficient information should be presented for the operator so that correct decisions can be made.

2.5.4 Integration Aspects

Physical integration based on standardized communication protocols ensure connectivity of devices and integration of controllers and operator stations into three network levels (Figure 2.3): 

- Real time field bus network communication on low level between devices and controllers,
- Real time control network connecting controllers and operator stations, and
- Office plant network to various office systems and information management systems. The last level opens up for satellite communication to land offices at ship operators or vendors. It is common to characterize the automation system by the number of I/Os and dividing the applications into low-end or high-end segments. For a drilling rig as shown in Figure 2.1 the number of I/O signals may be up to 10-15000. For a conventional supply vessel the I/O number is typically lower than 1.500 - 2.000 signals.

2.6 Dynamic Positioning System

In the 1960s systems for automatic control of horizontal position in addition to the course were developed. Such systems for simultaneously control of three horizontal motions (surge, sway and yaw motion) are today known as dynamic positioning (DP) systems. Description of DP systems including the early history can be found in Fay [74]. In the 1960s the first DP system was using single-input single-output PID control algorithms in combination with low-pass and/or notch filter. In the 1970s more advanced output control methods based on multivariable optimal control and Kalman filter theory was proposed. Later on nonlinear control was also applied, see Chapter 8 for details.

**Definition 2.1** A DP vessel is by the class societies e.g. Det Norske Veritas (DNV) [54], American Bureau of Shipping (ABS) [3] and Lloyd’s Register (LRS or Lloyd’s) [174], defined as a vessel that maintains its position and heading (fixed location or pre-determined track) exclusively by means of active thrusters. This is obtained either by installing tunnel thrusters in addition to the main screw(s), or by using azimuthing thrusters, which can produce thrust in different directions.
While in DP operated ships the thrusters are the sole source of station keeping, the assistance of thrusters are only complementary to the mooring system in the case of thruster assisted position mooring (PM) systems. Here, most of the position keeping is provided by a deployed anchor system. In severe environmental conditions the thrust assistance is used to minimize the vessel excursions and line tension by mainly increasing the damping in terms of velocity feedback control. For turret anchored ships, see Figures 2.4 and 2.5, without natural weather-vaning properties the thrusters are also used to automatic control of the heading, similarly to DP operated vessels. In this text, a marine positioning system is either defined as a dynamic positioning (DP) system or a thruster assisted position mooring (PM) system.

Figure 2.4: Marine operations in offshore oil and gas exploration.
DP systems have traditionally been a low-speed application, where the basic DP functionality is either to keep a fixed position and heading or to move slowly from one location to another. In addition specialized tracking functions for cable and pipe-layers, and remote operated vehicle (ROV) operations have been available. The traditional autopilot functionality has over the years become more sophisticated. Often a course correction function is available for correction of course set-point due to environmental disturbances and drifting, such that the vessel follows a straight line. Way-point tracking is used when a vessel is supposed to follow a pre-defined track e.g. defined by several way-point coordinates. The trend today is that typical high-speed operation functionality merges with the DP functionality, giving one unified system for all speed ranges and types of operations. Further research on hybrid control will enable this.

2.6.1 Position Reference Systems and Sensors

It is common to divide the different measurements used by a positioning system into position reference systems and sensor systems. The most commonly used position reference systems are:

- **Global Navigation Satellite Systems (GNSS):** The most commonly used navigation system for marine vessels is Navstar GPS, which is a US satellite navigation system with world coverage. An alternative system is the Russian system Glonass, which only covers certain regions of the globe, see Parkinson and Spiker [220]. An European system, Galileo, is currently under construction. For local area operations it is now possible to achieve meter accuracy by using Differential GPS (DGPS), and sub-decimeter accuracy by using Carrier Differential GPS (CDGPS). The development of wide area augmentation systems (WAAS) is expected to give meter accuracy across entire continents. When using a satellite navigation system, at least four satellites must be visible in order to compute a reliable position estimate (three for sea level navigation). If the ship is entering a shadowed region, and redundant signals are not available, there will be a loss of position measurement. Other causes for degraded position measurements (or in the worst case loss or measurement) are ionospheric disturbances, reflections in the water surface, etc. For DGPS the accuracy is typically within 1 meter radius with 95% probability. Recently, the forced signal
distortion (SA) on the GPS signal has been removed, such that the accuracy of the GPS itself without differential correction is dramatically improved. Depending of antennas and signal receivers, typical accuracy is within ±10 m. This opens up for even more cost effective navigation equipment within the field of marine positioning.

- **Hydroacoustic Position Reference (HPR) Systems:** By using one or several transponders located on fixed position on the seabed and one or several transducer mounted under the hull, the position of the vessel is measured. The accuracy of such systems depends on the water depth and the horizontal distance between the transponder and the transducer. There are different principles for performing measurements, where the most common used are super short baseline (SBL), super/ultra short baseline (S/USBL) or long baseline (LBL).

- **Taut Wire:** A *taut wire* system is used to measure the relative position of a floating vessel at rest. This system consists of a heavy load located at the sea floor. The load is connected to the vessel by a wire cord or heavy metal chain which is hold in constant tension by using a winch onboard the vessel. Furthermore the angles at the top and bottom of the wire are measured, and the length of the wire. Hence, the relative position \((x, y, z)\) can be computed by solving three geometric equations with three unknowns.

Other position reference systems can be microwave systems (ARTEMIS, MICRORAGER, MICRO FIX), Radiowave systems (SYLEDIS), optical systems (laser beams), and riser angle position measurement systems. The latter system is based on instrumentation on a marine riser attached to the vessel when performing drilling operations.

The sensor systems may comprise:

- **Gyrocompass and/or magnetic compass**, which measures the heading of the vessel.

- **Vertical reference unit (VRU),** which at minimum measures the vessel heave, roll and pitch motions. Angular velocities are also often available. One of the main functions for the VRU is to adjust the position measurements provided by GPS, hydroacoustic position reference (HPR) systems, etc. for roll and pitch motions. For deepwater DP operations the accuracy of the roll and pitch signals must be high providing accurate HPR position measurements.

- **An IMU typically contains gyros and accelerometers in 3-axes that can be used to measure the body-fixed accelerations in surge, sway and heave, the angular rates in roll, pitch and yaw and the corresponding Euler angles (Britting [35], Titterton and Weston [313]).** The IMU can be integrated in a filter (observer) with DGPS and HPR measurements for instance in order to produce accurate velocity estimates. In most cases only the IMU Euler angles are used in conjunction with DGPS. This is a minimum configuration since the Euler angles are needed in order to transform the measured GPS position corresponding to the GPS antenna down to the vessel fixed coordinate system usually located in the centerline of the ship.

- **Wind sensors,** which measures the wind velocity and direction relative to the vessel.

- **Draft sensors** (used for vessels which is operated over a wide range of drafts).
• Environmental sensors: wave sensors (significant wave height, direction, peak frequency),
current sensors (velocity and direction at sea surface and at different depths). Environmental sensors are not a class requirement. These sensors are quite common on the most sophisticated offshore installation.

• Other sensors dependent on type of operation, e.g. for pipe layers pipe tension is also measured and utilized by the DP system.

In many installations redundant measurements are available, and the number and types of measurements required are specified by certain class rules, see for example DNV [54]. Redundant measurements increases the safety and availability of the positioning system.

The heading of the vessel is usually measured by one or several gyrocompasses, which dynamically are accurate. During rapid turning operations, however, the gyros will produce a steady-state offset which gradually decreases to zero when the course is constant again. By using two GPS antennas, the heading can be measured even more accurately, without the drifting effect. In the case of anchored vessels, measurements of line length and tension are usually interfaced to the PM control system. Any positioning system requires measurement of the vessel position and heading. Wind sensors, measuring the wind speed and relative direction, are commonly used for wind feedforward control. In most commercial systems measurements of surge and sway velocities are not available with sufficient accuracy and must be estimated on-line. A guidance rule by DNV [54] is that the accuracy of the position reference data is generally to be within a radius of 2% of the water depth for bottom-based systems, and within a radius of 3 meters for surface-based systems.

A minimum sensor and navigation configuration for a DP system typically consist of at least one position reference system, one gyro compass, one VRU for roll and pitch measurement and one wind sensor. The redundancy of the DP system can of course be increased by multiple measurement devices. However, one should also consider using systems based on different measurement principles, giving full redundancy not only in hardware configuration.

The trend today is integration between sensor systems and position reference systems. Over the last years the price of accurate inertial motion units (IMU’s) have been reduced. This trend opens for integration between inertial navigation systems (which are based on the IMU sensor units) and other position reference systems e.g. GPS.

2.6.2 Modes of Operation

The DP and PM control functionality are closely connected to the surge, sway and yaw degrees of freedom (DOF), which can be viewed as independent of the actual thruster configuration, as long as there are enough thrust capacity to fulfill the force and moment demands. The various DOFs can individually be controlled in the following modes of operation:

• **Manual Control:** By using a joystick and a rotation knob the operator of the ship can generate force set-points in surge and sway and a moment set-point in yaw for manual control of the ship.

• **Damping Control:** Feedback is produced from estimated low-frequency vessel velocities and the objective is to regulate the vessel in the specific axis towards zero. This mode is especially applicable for anchored vessels, where effective damping will reduce possible large oscillatory motions, experienced about the resonance period of the anchored vessel,
and thus reduce the stress on the mooring system. Damping is also useful in DP for obtaining a smooth transition between transit speed and fixed position operations.

- **Set-Point Control:** Feedback is produced from both estimated low-frequency velocities and position/yaw angle. The objective is to keep the actual axis at the specified set-point position or heading.

- **Tracking Control:** The vessel tracks a reference trajectory which is computed from the old to the new position or heading set-point.

The most common DP operation is naturally set-point control in all three axes, often referred to as *station keeping*. Other fully automatic modes of operation can be station keeping with weather optimal positioning, and roll- pitch damping. In the first case the vessel automatically tends to the heading where the effect of the environmental loads are minimized. In the latter case the roll and pitch motion is suppressed by proper action of the horizontal thrust components. A combination between different modes are common, e.g. semi-automatic mode where a joystick is used for manual surge and sway control, while the heading is automatically controlled. Other tailor made DP functions exist for tracking operations like cable and pipe laying, ROV operations, etc.

The combination of damping control in surge and sway and set-point control of heading is often used in PM systems, especially in bad weather. Moreover, the vessel will tend to an equilibrium position where the mean environmental forces are balanced by the mooring forces. Assuming a proper heading set-point, this equilibrium position will be optimal with respect to the thrust usage and the fuel consumption. For turret-moored ships automatic heading control is often the most important function. By keeping the heading optimally against the weather the effect of the environmental loads, and thereby the stress on the mooring system, will be minimized.

### 2.6.3 Functionality and Modules

The various DP vendors may differ in design methods. However, the basic DP functionality are more or less based on the same principles, as outlined in Figure 2.6.
Signal processing

All signals from external sensors should be thoroughly analyzed and checked in a separate signal processing module. This comprises testing of the individual signals and sensor signal voting and weighting when redundant measurements are available. The individual signal quality verification should include tests for signal range and variance, frozen signals and signal wild points. If an erroneous signal is detected, the measurement is rejected and not used by the positioning system. The weighted signals from each sensor group should not contain any steps or discontinuities when utilized further in the system ensuring a safe operation.

Vessel observer

Filtering and state estimation are important features of a positioning system. In most cases today, accurate measurements of the vessel velocities are not available. Hence, estimates of the velocities must be computed from noisy position and heading measurements through a state observer. The position and heading measurements are corrupted with colored noise, mainly caused by wind, waves and ocean currents. However, only the slowly-varying disturbances should be counteracted by the propulsion system, whereas the oscillatory motion due to the waves (1st-order wave-loads) should not enter the feedback loop. The so-called wave-frequency modulation of the thrusters will cause unnecessary wear and tear of the propulsion equipment. In the observer so-called wave filtering techniques are used, which separates the position and heading measurements into a low-frequency (LF) and a wave-frequency (WF) part. The estimated LF position, heading and velocities are utilized by the feedback controller. The observer is also needed when the position or heading measurements temporarily are unavailable. This situation is called dead reckoning, and in this case the predicted estimates from the observer are used in the control loop. Another feature of the observer is that it estimates the unmodeled and unmeasured slowly-varying forces and moments, mainly due to second-order wave loads and ocean current.
Controller logics

The positioning system can be operated in different modes of operation, see Section 2.6.2. All kind of internal system status handling and mode transitions, model adaptation etc. are governed by the controller logic. This includes smooth transitions between the different modes of operation, issue alarm and warnings, and operator interactions.

Feedback control law

The positioning controllers are often of the PD type (multivariable or decoupled in surge, sway and yaw), where feedback is produced from estimated LF position and heading deviations and estimated LF velocities. The underlying control methods may, however, vary. Traditionally, decoupled controllers and linear quadratic controllers have been popular. In addition to the PD part, integral action is needed to compensate for the static (or slowly-varying) part of the environmental loads. The controller should be optimized with respect to positioning accuracy, fuel consumption, and wear and tear of the propulsion system. In the design of positioning controller, the control inputs are forces in surge, sway and moment in yaw. In the context of positioning systems, this may be regarded as high-level control, since the actual control inputs are shaft speed (RPM control) or the pitching of the propeller blades (pitch control), which indirectly controls the developed force. In the case of azimuth thrusters, the direction of each thrust device are additional control inputs.

Guidance system and reference trajectories

In tracking operations, where the ship moves from one position and heading to another, a reference model is needed for achieving a smooth transition. In the most basic case the operator specifies a new desired position and heading, and a reference model generates smooth reference trajectories/paths for the vessel to follow. A more advanced guidance system involves way-point tracking functionality, optimal path planning and weather routing for long distance sailing. The guidance system could be interfaced to electric map systems.

Feedforward control laws

The most common feedforward control term is wind feedforward. Based on measurements of wind speed and direction, estimates of wind forces and moment acting on the ship are computed. As a consequence, a fast disturbance rejection with respect to varying wind loads can be obtained. In order to improve the performance of the system during tracking operations, a reference feedforward is computed. This is done by using a model of the ship dynamics, reference accelerations and velocities, given by the reference model. In PM systems wind feedforward is normally enabled in yaw only, since stationary wind loads in surge and sway should be compensated by the mooring system. In addition, a line break detection algorithm is monitoring the line tension signals and the corresponding vessel motions, in order to automatically detect break in the anchor lines. When a line break is detected, the line break feedforward controller will be activated in order to have the thrusters compensating for the lost forces and moment produced by the broken line. This will ease the load in the surrounding lines and thus preventing yet another line break.
**Thrust allocation**

The high-level feedback and feedforward controllers compute commanded forces in surge and sway and moment in yaw. The *thrust allocation* module computes the corresponding force and direction commands to each thrust device. The low-level thruster controllers will then control the propeller pitch, speed, torque, and power satisfying the desired thrust demands. The thrust allocation algorithm should be optimized for fuel consumption, wear and tear of the thruster devices and for obtaining the commanded thrust in surge, sway and yaw. In addition, the function should take into account saturation of the $\frac{\rho \pi \mu}{\text{pitch}}$ inputs and forbidden directional sectors. The thrust allocation module is also the main link between the positioning system and the power management system (PMS). The positioning system has a very high priority as a power consumer. In any case, the thrust allocation must handle power limitation of the thrusters in order to avoid power system overload or blackout. The thrust allocation module receives continuously updated inputs from the PMS about available power and prevailing power plant configuration with status on the bus ties and generators. This should prevent power black-out and undesired load shedding of other important power consumers.

**Model adaptation**

The parameters in the mathematical model describing the vessel dynamics will vary with different operational and environmental conditions. In a model-based observer and controller design, the positioning system should automatically provide the necessary corrections of the vessel model and controller gains subject to changes in vessel draught, wind area and variations in the sea state. This can be obtained either by gain-scheduling techniques or continuously by using non-linear and adaptive formulations. In addition other adaptive control and estimation methods may be applied, either run in batches or processing on-line.

### 2.6.4 Advisory and Surveillance Systems

Use of advisory systems for diagnostics, simulation and analysis of future operational requirements, subject to varying environmental and operational conditions, becomes of increasing importance for optimal operational planning. Such systems are integrated with the positioning system and typical features are described below:

- **DP and PM vessel motion simulators.** Such simulators include mathematical models of the environment and the vessel motion, and the operator can simulate the performance of the positioning system, either using the prevailing environmental conditions or specify any environmental condition. In addition, different types of failure scenarios can be simulated, such as power or thruster failure, drive- and drift off. In PM systems failures in one or several lines can be simulated as well.

- **Consequence Analysis.** This is an advanced version of a position capability analysis, which continuously verifies that the vessel is capable of keeping position and heading for different failure scenarios during the prevailing conditions. This can be loss of one or several thrusters, one engine room or mooring lines (if applicable). In many cases a similar off-line version is also available, where *any* environmental condition, operation or failure situation can be simulated, by request from the operator.
2.6.5 DP Capability

The main purpose of a DP system is to keep position (and heading) within a certain excursion limits within a specified weather window, or so-called design environment. In order to meet the designed positioning capability, the system components should be reliable and the necessary redundancy requirements should be met. During the design phase it is important to verify that the amount of power and thrust capacity installed on a vessel will provide the necessary holding capacity. This can be done either by static or dynamic analysis. In a static analysis only the mean slowly varying forces due to wind, current and waves are considered. The items of data required in such a study include:

- Main vessel particulars, such as displacement, length, breadth and operating draught.
- Directional dependent wind, current and wave-drift coefficients, from which the corresponding forces and moments can be computed.
- The maximum environmental conditions in which the vessel should operate in dynamic positioning (wind speed, significant wave height and current speed). Important parameters are dominating wave period and the statistical description of the waves, usually described by wave spectrum formulations such as the Bretscheider spectrum, the Pierson-Moskowitz spectrum or the Joint North Sea Wave Project (JONSWAP) spectrum.

A rule of thumb is that the most loaded thruster should not use more than 80% of the maximum thrust in the design environment to compensate for the static loads, (API). The 20% margin is then left for compensation of dynamic variations. The results of such a static analysis can be presented as a capability plot (Figure 2.7). The sectors about 120° – 133° and 240° – 250° where the "butterfly-shape" curvature crosses the unity circle, indicate lack of thrust capability to maintain the position and the heading for the environmental loads acting in these directions. The environmental load data is given to the right on Figure 2.7. The butterfly-shape curvature results from rotating the environmental loads 360° about the vessel. The load directions given to the right are the prevailing directions from a real DP operation. The static analysis with 20% margin is often too conservative. This suggests that the capability analysis should be complemented with dynamic (time domain) simulations, where the thrust allocation, the inherent thruster dynamics and dynamic thruster losses, forbidden azimuthing sectors and the whole control loop is taken into account in the final verification. In the traditional capability simulations the effect of power limitation is often neglected, which in fact often is the most limiting factor in failure situations. A capability analysis with power limitation is also important in the design of the power plant; how many power busses, amount of power generation on each bus, which thruster to be connected to the different busses, etc.

2.7 Power and Energy Management

In a system of electrical power installations, vessel and process automation system, DP system, and the other parts of the automation system controls their parts of the power system, e.g. the DP system controls the thruster drives, the off-loading control system use cargo pump drives, the process control system interacts with the compressors and cooling/heating systems etc. The interconnecting point for all the installed power equipment is the power distribution system. By starting and inrush transients, load variations, and network disturbances from harmonic effects,
the load and generators are interacting and influencing each other. Optimum operation and control of the power system is essential for safe operation with a minimum of fuel consumption. As it is the energy control system (power/energy management system – PMS/EMS), which monitors and has the overall control functionality of the power system, it will be the integrating element in a totally integrated power, automation and DP system.

The purpose of the PMS is to ensure that there is sufficient available power for the actual operating condition. This is obtained by monitoring the load and status of the generator sets and the power system. If the available power becomes too small, either due to increased load or fault in a running generator set, the PMS will automatically start the next generator set in the start sequence. A PMS can also have extended functionality by monitoring and control of the energy flow in a way that utilizes the installed and running equipment with optimum fuel efficiency. Such systems can be EMS. Energy management is a new approach to control and monitor the energy flow in marine, oil and gas systems. The EMS extends the concept of power management in the direction of controlling and coordinating the energy generation and consumption. In addition to optimize the instantaneous power flow, the historical energy usage and future energy demands are considered. EMS will then be the integrating element in a totally integrated power, automation and DP system. For PMS and EMS, the main functions can be grouped in Sørensen and Ådnanes [301]:

- **Power generation management**: Overall control with frequency and voltage monitoring with active and passive load sharing monitoring and possibly control, and load dependent start and stop of generator sets. Since control logic and interlocking functions are a significant part of the power system switchboard design, the functionally of these systems must be coordinated.

- **Load management**: Load power monitoring and coordination of power limitation functions in other systems, load shedding and start interlock of heavy consumers based on available
power monitoring,

- Distribution management: Configuration and sequence control for reconfiguration of the power distribution system. The distribution system should be configured to fit the requirements in the actual operational mode for the vessel.

The new generation production vessels and also drill ships/rigs have a complex power system configuration with advanced protection and relaying philosophies. There are close connections between the functional design and performance of the PMS and the power protection system functions. It is a challenge for involved parties to obtain an optimal and functional solution with several suppliers involved and a yard being responsible for all the coordination.

### 2.7.1 Blackout Restoration

Blackout of the power generating system is the most severe fault that can happen in an electric propulsion system. Should a blackout occur, and it does unfortunately happen from time to time, there will normally be required to have a system for sequence control of start-up and reconfiguration of the power system. This is implemented at the system control level, and includes sequences for starting and synchronizing generator sets and loads. There will normally also be a set of predefined operation modes, e.g. transit mode, station keeping mode, maneuvering mode, etc. with automatic sequence control for power system reconfiguration. A typical sequence for:

- Automatic start and connection of emergency generator control. Separate control system - not part of the overall vessel control system.
- Automatic start/reconnection of standby generator(s).
- Reconnection of transformer breakers, distribution breakers.
- Tie breakers remain open if tripped.
- Restart of fuel and cooling pumps.
- Restart of other pumps and fans in sequence.
- Restart of thrusters, if required.

### 2.7.2 Load Reduction and Blackout Prevention

The diesel-electrical power system has better performance and fuel economy if the PMS controls the number of running engines in the power plant to match the load demand with high loading of the diesel engine. The optimal load with respect to fuel consumption, tear and wear, and maintenance is typically about 85% of maximum continuous rating (MCR). With a high load at the running engines, the system gets more vulnerable to faults in the system, such as a sudden trip of one diesel engine generator. The remaining, healthy engines will experience a step load increase and possible overloading, and in worst case, under frequency trip unless the functionality in the system reduces the load power in accordance with the generating capacity. In a modern drilling vessel, this load reduction and blackout prevention function is distributed and handled by several subsystems like:

- Power management’s load management and blackout prevention functions.
Figure 2.8: Regulation time constants for power reduction, with maximum response time in the order of 500ms (Illustrative only).

- DP system’s power limitation functions.
- Thruster drive’s load reduction and load phase back functions.
- Drilling drive’s load reduction and load phase back functions.

The ability to withstand such faults is also highly depending on the design and engineering methods and solutions, such as:

- Load capability and dynamics of the diesel engine.
- Governor and AVR configuration and settings.
- Generator and switchboard protection relay settings.
- Critical equipment should be fault-tolerant with loss of power ride-through functionality.

In DP vessels with high efficient speed controlled fixed pitch thrusters, the total load under normal operation is so low that the power plant runs optimal with few, often only two running diesel engines. In order to utilize this saving potential, the challenge is to design and tune the system to be capable of handling fault scenarios within a time frame not compromising the stability of the power supply.

Figure 2.8 shows, for illustration, the diesel engines capability to maintain the frequency for the load step associated with the loss of a parallel run engine. In typical installations, it has been seen that the actions of load reduction and blackout prevention must be effective within less than 500ms in order to not compromise the power system stability and limit the flexibility of operation. Several solutions are in use and the operators and owners have different preferences. However, some common conclusions can be made on what is typically required for the blackout prevention functionality:
• **Thruster and thruster drives:** Variable speed FPP thrusters must have a load reduction scheme, either monitoring the network frequency and/or receiving a fast load reduction signal from the PMS, either as a power phase-back signal, maximum power limitation signal, or – if well coordinated – fast RPM reference reduction. In order to avoid instabilities in the network frequency, the load reduction should be as precise as possible in order to dampen potential oscillations. Fixed speed CPP thrusters do not have fast enough response time for blackout prevention. These must be included in the power management’s load shedding scheme.

• **Drilling drives:** Similar to the requirements of the thruster drives, with built-in priorities for the individual drilling drives.

• **PMS:** By class requirements, the PMS must include blackout prevention with load reduction/load shedding functionality. It was observed earlier, that the response time in this system was too long to obtain the desired level of fault-tolerance without a fast acting, stand-alone load reduction scheme in the thruster drives. With the knowledge of today, this has been claimed solved by use of fast acting, and possibly event-trigged load reduction algorithms.

• **DP system:** The DP system is also equipped with a power limitation function, normally based on a permitted maximum power consumption signal from the PMS. Generally, this has shown to be effective in avoiding overloading of the running plant, but not fast enough to handle faults and loss of diesel-generator sets. Of importance is also that the power limitation in manual and joystick control of the thrusters.

Based on experience, it is recommended that all load reduction and blackout prevention functions described above are installed and well coordinated, tuned and tested during commissioning and sea trial. Also, the need for retuning and testing must also be considered after modifications in the installation that may affect the coordination.

A typical coordination diagram is shown in Figure 2.9. Auto start and auto stop limits shows the level and time settings for load dependent automatic start and shutdown of the engines. Available power will under normal operations be within these limits. Upon faults, and sudden loss of engine, the available power is being reduced. The power reduction functions of the DP can be distinguished between critical or non-critical situations, allowing the DP to take all available power after possible load reduction and load shedding of non-essential consumers or consumers with lower priority.

### 2.7.3 Diesel Engine Governor and AVR Fault Tolerance

Although the rules and regulations have allowed for DP operations with closed bus ties in the electrical power system and one commonly connected power plant, the practices were until mid 1990s to split the network into two or more separated sections in DP 2 or 3 operations. With the developments on switchboard protection relays, and thruster drives together with new PMS with more precise and faster load reduction and blackout prevention functions, this has changed and it is now more common to operate with normally closed bus tie breakers and also with closed ring main bus. This development is motivated by the benefits of better and more flexible utilization of the installed generating capacity and to gain an improved fuel economy. The fuel savings for optimized loading of running engines have shown to be significant.
In such systems, it has been observed that under some load and operating conditions, certain faults in the governors and automatic voltage regulators, AVRs, can be difficult to identify with a regular protection scheme. In worst case, faults have shown to interfere with healthy equipment causing undesired shutdowns, and in some cases, even blackout. Developments in digital generator protection relays with multifunction and programmable protection logics have enabled a possibility to combine protection functions in new manners, and to include logic and algebraic functionality into the protection scheme. This has significantly improved the protection relay’s ability to detect governor and AVR faults, and improved the system’s fault tolerance to such faults. Figure 2.10 shows a possible monitoring scheme for AVR, including correlation functions of multiple variable and voting functions for multiple gensets. Such functionalities can be installed in the modern, programmable multifunction generator protection relays, or in separate logic controllers for retrofit and upgrades.

2.8 Maritime Industrial IT

We will in this chapter also shortly address aspects related to the information flow between the real time systems ensuring on-line control and the management systems optimizing the operations and the business processes. This includes fleet allocation related to transport logistics, supply chain management, monitoring and diagnostics of technical condition of equipment and systems.

As reported in Rensvik et al. [228] systems for operational management such as condition monitoring and diagnostics systems, supply chain management systems, enterprise management systems, etc. have increased the possibility to improve operational performance, productivity and life cycle optimization of the assets related to operation of the installations.

Lately, the automation vendors, mainly in land-based industry sectors, have started the next
step to physically and functionally integrate the real time control systems with the operational management systems. This has been denoted as industrial IT. The introduction of industrial IT into marine applications has yet only started, and is still an area of research and development. For vendors and ship operators it is a challenge to take out the potential this shift of technology gives.

Maritime industrial IT solutions for the various marine market segments will be dependent on type of trade and charter, vessel complexity, safety and availability requirements, size of fleet, etc. Concerning vessel automation we will here focus on ships and vessels characterized as advanced and specialized with high number of input/outputs (I/O) and rather complicated operational functions. This is denoted as the high-end market; see also Figure 2.11. Examples are ships (Figure 2.4) and rigs (Figure 2.1) for oil and gas exploration and exploitation, passenger and cruise vessels. Typical applications in the offshore market are service vessels, drilling rigs and ships, shuttle tankers, cable and pipe layers, floating production off-loading and storage units (FPSOs), crane and heavy lift vessels, geological survey vessels and multi-purpose vessels.

As a part of enabling industrial IT solutions some vendors have installed condition monitoring and control functions locally on the power equipment and field devices as indicated in Figure 2.3 with the possibility for remote monitoring and diagnostics.

In many installations redundant systems are available, and the number and types of measurements required are specified by certain class rules. Physical integration based on standardized communication protocols ensure connectivity of devices and integration of controllers and operator stations into three network levels (Figure 2.3) real time field bus network communication on low level between devices and controllers, real time control network connecting controllers and operator stations, and office plant network to various office systems and information management systems. The last level opens up for satellite communication to land offices at ship operators or vendors, see also Figure 2.12. Industrial IT is supposed to increase the integration of vessel plant data with the business management systems ensuring optimized asset management and operation of each vessel in particular and the whole fleet on corporate level, see Fig.
Figure 2.11: Low-end and high-end market segments within automation and ship management.

Figure 2.12: Distributed information systems ships – shore for flexible information flow and sharing.
As cost on the vessel-to-land satellite communication is reduced (Figure 2.14) and the maritime information technology architecture is improved this kind of information flow is expected to be working seamless in real time, as opposed today, where a limited amount of data is transferred at discrete events.

Up to now integrated automation systems have been proprietary with a limited number of vendors. However, in the automation industry it is a trend towards openness in communication protocols and network. How this will influence on the technology solutions and responsibilities for multi-vendor integration systems is still a subject for discussion.

Condition monitoring and RCM (Reliability Centered Maintenance) are well known concepts in many industries. These concepts have now also been incorporated in ship maintenance management systems. Today, ships in many trades have only very short loading and discharging periods in port. This means that maintenance that only can be performed in port including
flag state and class surveys have to be carried out during these short periods to avoid downtime. To improve such a harmonization, analysis and planning methods have to be available to monitor the status of the vessel, to schedule surveys and to predict the future state of the ship based on frequent reporting and continues monitoring of technical condition for structures and equipment (Figure 2.15). This concept can further be developed into a “continues” risk analysis based on this technical condition information, and adapted to highlight risk equipment related to the different operational mode of the ship, i.e. a way to enhance safety measures. Frequent tools in technical monitoring is the use of signal processing techniques such as spectrum analysis and other statistical methods. Examples are acceleration measurements for vibration analysis, temperature measurements for overheating, etc. Lately, the use of observers for state estimation may also be used.

2.9 Rules and Regulations

2.9.1 Class Requirements

In many cases it is up to the owners to decide which type of classification (or to seek for class approval at all) for the positioning system installed. Three examples of class societies that have rules for classification of DP systems are DNV [54], ABS [3] and LRS [174]. The International Marine Organization (IMO) has also developed Guidelines for Dynamic Positioning, in order to provide an international standard for DP systems on all types of new vessels. The purpose of the Guidelines and class rules is to recommend design criteria, necessary equipment, operating requirements, and a test and documentation system for DP systems to reduce the risk to personnel, the vessel itself, other vessels and structures, sub-sea installations and the environment.

Figure 2.15: Technical condition philosophy.
while operating under dynamic positioning control. Taking into consideration that DP operated vessels often operate in different parts of the world, such a standardization provides a useful tool for the different Costal states to specify the local rules and regulations, defining levels of safety requirements, requirements for redundancy and operations for DP vessels. For moored vessels using automatic thruster assistance, IMO does not have any guidelines. DNV has the class notation called POSMOOR ATA, DNV [55] and Lloyd’s Register of Shipping has the notation PM or PM T1, LRS. More general document for analysis, design and evaluation of moored, floating units is published by the American Petroleum Institute [11]. Here some guidelines for DP systems are also included.

The requirements for hardware and software on DP systems are closely connected to the level of redundancy, which is defined as:

**Definition 2.2 (Redundancy)** Redundancy means ability of a component or system to maintain or restore its function when a single fault has occurred. This property can be obtained by installation of multiple components, systems, or alternative means of performing a function.

A DP system consists of components and systems acting together to achieve sufficiently reliable positioning keeping capability. The necessary reliability of such systems is determined by the consequence of a loss of position keeping capability. The larger the consequence, the more reliable the DP system should be. To achieve this philosophy the requirements have been grouped into three different equipment classes. The equipment class depends on the specific DP operation, which may be governed by Costal state rules and regulations or in agreement between the DP operator company and their customers. A short description of the different classes is given below.

**Class 1** For equipment class 1, loss of position may occur in the event of a single fault, e.g. the DP control system need not to be redundant.

**Class 2** For equipment class 2, loss of position is not to occur in the event of a single fault in any active component or system. The DP control system must have redundancy in all active components, e.g. the hardware must consist of at least two independent computer systems with self-checking routines and redundant data transfer arrangements and plant interfaces. At least three independent position reference systems and three sensor systems for vertical motion measurement, three gyrocompasses and three wind sensors.

**Class 3** Same as DP class 2, with additional requirements on redundancy in technical design and physical arrangement.

Class 2 or 3 system should include the function ”Consequence analysis”, which continuously verifies that the vessel will remain in position even if the worst single failure occurs. The IMO Guidelines also specifies relationship between equipment class and type of operation. DP drilling operations and production of hydrocarbons, for instance, requires equipment Class 3, according to IMO. In Table 2.1 the coherence for DP class notations by DNV, LRS, ABS and IMO are shown.

In Figure 2.16 DNV [54] has listed up the minimum requirements for DP class notation.
Table 2.1: Mapping table of DP class notation by IMO, DNV, Lloyds and ABS.

<table>
<thead>
<tr>
<th>IMO Equipment Class</th>
<th>DNV</th>
<th>Lloyds</th>
<th>ABS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Not applicable</td>
<td>Dynpos AUTS</td>
<td>DP CM</td>
<td>DPS-0</td>
</tr>
<tr>
<td>Class 1</td>
<td>Dynpos AUT</td>
<td>DP AM</td>
<td>DPS-1</td>
</tr>
<tr>
<td>Class 2</td>
<td>Dynpos AUTR</td>
<td>DP AA</td>
<td>DPS-2</td>
</tr>
<tr>
<td>Class 3</td>
<td>Dynpos AUTRO</td>
<td>DP AAA</td>
<td>DPS-3</td>
</tr>
</tbody>
</table>

Figure 2.16: Minimum DNV requirements (2010) for DP class notation, courtesy by DNV.
2.9.2 Reliability and Redundancy

From a safety point of view a DP system can be viewed as four different sub-systems. Each sub-system can then be further split recursively into sub-sub systems. For instance:

- Level 1: Power system. Level 2: Power generation, power distribution, drives, etc.
- Level 1: Propulsion system. Level 2: Main screw, tunnel thrusters, azimuth thrusters.
- Level 1: Positioning control system. Level 2: Computer and I/O, Operator HMI, UPS, Operator interaction.

Starting from the bottom, the reliability of each sub-component can be specified and depending on the level of redundancy, the reliability and availability of the total system can be computed using statistical methods. Each component can be characterized by:

- Failure rate \( \lambda \), defined Maximum number of failures per million hours.
- Mean time between failures (MTBF), for one component given by

\[
MTBF = \frac{1}{\lambda}
\]

- Total down time, \( T_d \), of a component, including mean time to repair (MTTR).
- Availability, \( A \), defined as

\[
A = \frac{MTBF}{MTBF + T_d}
\]

The above characteristics are specified for each component or each level in a reliability analysis and will at the top level characterize the whole plant.

The redundancy concept of the electrical power and propulsion plant will be based on the required ability for maneuvering and propulsion after faults in the system. In commercial vessels, these requirements are determined by national and international legislations, and specified by the classification societies by the different class notations. A typical redundancy diagram for a DP class 2/3 drilling rig is indicated in Figure 2.17. Each block represents a part of the system which is susceptible to a single failure.

At higher level of controls the redundancy concept is different and achieved by the duplication of the control systems in hot back-up configuration, as seen in Figure 2.18 for a typical DP 3 drilling rig.

The vessel control system must follow the same redundancy and segregation principles as the electric system. Principally, this is achieved by designing the vessel control structure as a mirror image of the electrical power plant, as shown in Figure 2.19.
2.9.3 Failure Analysis

The cost of making design changes during the initial project phase is small, but as the project progresses the cost of design changes increases significantly. Minor changes in a system can cause project delays and huge additional costs during the commissioning and sea trials. During the whole design phase of newbuildings, the reliability of the total system can be thoroughly investigated by different reliability methodologies, in order to detect system design errors and thus minimize the risk. Such analysis of DP system and its subsystems is not required by the class societies or the IMO Guidelines. However, the Coastal states, oil companies or customers may require it, in addition to classification approval. There are different methods available for assessing the reliability of such complex systems.

One common methodology is the Failure Mode and Effect Analysis (FMEA). This is a qualitative reliability technique for systematically analyzing each possible failure mode within a system and identifying the resulting effect on that system, the mission and personnel. This analysis can be extended by a criticality analysis (CA), a quantitative procedure which ranks failure modes by their probability and consequence.

2.10 Simulation

For years numerical simulators have been used as tools in system design and analysis, both in academia and in the industry. The simulator may use models of various fidelity to reconstruct the real physical properties of a dynamic system, Sørensen et al. [298]. For control system
Figure 2.18: A total integrated vessel control system for drilling vessels (Courtesy of Kongsberg Maritime).
Figure 2.19: Mirror image design of the vessel control system ensures to achieve a unified redundancy and segregation philosophy.
design and testing purposes it is convenient to develop a real-time system simulator, Figure 2.20. In order to operate in real-time often simplified or equivalent models of fast dynamic systems like power electronics, complex systems like multi-dimensional finite element method (FEM) models of structures and panel methods of hydrodynamics must be used. Model reduction and simplifications must be done with care such that important structural information and properties of the dynamic system are not lost.

At NTNU a marine system simulator illustrated in Figure 2.21, Marine Systems Simulator (MSS, [195]), based on MATLAB/SIMULINK. MSS integrates the disciplines hydrodynamics, structural mechanics, marine machinery, electric power generation and distribution, navigation and automatic control of marine vessels. The main purpose of MSS is to improve the accumulation and reuse of knowledge and thereby the quality of the education and research. The simulator will be continuously developed by students and researchers, and will serve a diversity of applications. This necessitates a modular structure, in which each module is a self-contained unit with a well-defined interface and functionality.

### 2.10.1 Simulator Structure

The core of the simulator is the *process plant models or simulation models* which give the necessary detailed description of the vessel dynamics, systems and components and its surroundings, see Figure 2.21. The other main parts of the simulator are the control systems interfaced with the sensor and actuator modules. The control systems may for instance be DP system, thruster assisted position-mooring system, tracking controllers and autopilots, local thrust controllers, PMS, crane control systems, etc. MSS is made partly as open-source SW.

### 2.10.2 Module Hierarchy

Depending on the development stage and required accuracy of the operation and application studied, a hierarchy in module complexity is allowed. For any application, several modules of varying technical complexity, built-in functionality and release levels may exist. These should all cover the same basic functionality and have the same interfaces, and may therefore easily
be interchanged. Figure 2.22 shows a module hierarchy example, where module complexity is plotted versus release level and application.

2.10.3 Hardware and Software Platform

The simulator is currently being developed in a MATLAB/SIMULINK environment on the Windows PC platform. SIMULINK was chosen for running the main simulation loop because of its flexibility towards several programming languages. Applications written in MATLAB, C, C++, and Fortran may easily be linked to the simulation by use of S-functions. This is convenient for generating simulator modules from existing code, and makes development of new modules more user-friendly.

2.10.4 Hardware-In-the-Loop Testing

As the DP vessels become more demanding and complex, safety, reliability and integration aspects with the navigation system, power plant, vessel automation, propulsion system and other consumers become more important. In order to reduce these risks, regulatory bodies, class societies and independent consultants have been continuously addressing advances in rules and regulations and testing and verification methodologies. In this context the safety and verification regime for DP systems may be seen as an example to be followed for other mission critical control systems as well.

The successful operation of DP vessels such depends more and more on advanced integrated functionality of software-based control systems. Consequently, software related problems, often in conjunction with hardware and/or human errors, may lead to vessel construction delays, downtime during operation, reduced income for clients, increased cost, and reduced safety. In
order to reduce these risks, independent third party Hardware-In-the-loop (HIL) simulator testing has recently been applied for extensive software testing and verification of DP systems on several offshore vessels. In the work of Johansen et al. ([135], [136]), Johansen and Sørensen [137] and Smogeli, [267] the concept of HIL testing is described, and the experiences and findings statistics are reported from HIL testing of DP computer systems, power management systems and steering, propulsion and thruster control systems on drilling vessels, offshore service and construction vessels, and shuttle tankers. The main idea is testing and verification of the computer software using a vessel specific simulator (Figure 2.23) capable of simulating the dynamic response of the vessel, thruster and propulsion system, sensors, position reference systems, power generation, distribution, main consumers, and other relevant equipment (Sørensen et al., [298]). The simulator is connected via network or bus interfaces to the targeted control system such that all relevant feedback and command signals are simulated. In order to achieve the test objective, the simulator is capable of simulating a wide range of realistic scenarios defined by operational modes, operational tasks and single, common mode and multiple failure modes in order to verify correct functionality and performance during normal, abnormal and faulty conditions.

HIL testing may be conducted in several phases of a new-building or retrofit, where the first phase is usually an extensive software test conducted at factory or a lab facility. By using HIL simulator technology a virtual sea trial with thorough testing is conducted before the vessel is built. The objective is fully functional and failure testing of the software before the commissioning and integration starts, ensuring that the software will be more finalized and ready for commissioning. Follow-up system and integration testing is normally conducted during commissioning, and a final verification of the integrated functionality is conduced onboard the vessel at the end of commissioning.
Figure 2.23: HIL Testing.
Chapter 3

Maritime Electrical Installations and Diesel Electric Propulsion

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3.1 Introduction

3.1.1 Scope and Objectives

Electrical installations are present in any ship, from powering of communication and navigation equipment, alarm and monitoring system, running of motors for pumps, fans or winches, to high power installation for electric propulsion.

Electric propulsion is an emerging area where various competence areas meet. Successful solutions for vessels with electric propulsion are found in environments where naval architects, hydrodynamic and propulsion engineers, and electrical engineering expertise cooperate under constructional, operational, and economical considerations. Optimized design and compromises can only be achieved with a common concept language and mutual understanding of the different subjects.

The objective of this section is to give an introduction to electro-technology in general, and put special emphasis on installations for electric propulsion. It is the aim to give engineers with marine competence and background the necessary understanding of the most important electro-technical subjects used in design and configuration of ships with electric propulsion.

After an introductory review of the history of electric propulsion in Section 3.1 and application areas of electric propulsion in Section 3.2, an overview of the electric power system in Section 3.3 and its associated control systems in Section 3.4 follows before the main characteristics of the electric propulsion drives are presented in Section 3.5. Important design and engineering considerations are discussed in Sections 3.6 and 3.7 before ending by showing typical arrangements in Section 3.8 by use of single line drawings of the electrical installations in some important applications.
3.1.2 Motivations for Electric Propulsion

The concept of electric propulsion is not new, the idea originated more than 100 years ago. However, with the possibility to control electrical motors with variable speed in a large power range with compact, reliable and cost-competitive solutions, the use of electrical propulsion has emerged in new application areas during the 80’s and 90’s.

Electric propulsion with gas turbine or diesel engine driven power generation is used in hundreds of ships of various types and in a large variety of configurations. Installed electric propulsion power in merchant marine vessels was in 2002 in the range of 6-7 GW (Gigawatt), in addition to a substantial installation in both submarine and surface war ship applications.

By introduction of azimuthing thrusters and podded thrust units, propulsion configurations for transit, maneuvering and station keeping have in several types of vessels merged in order to utilize installed thrust units optimally for transit, maneuvering and dynamically positioning (dynamic positioning - DP).

At present, electric propulsion is applied mainly in following type of ships: Cruise vessels, ferries, DP drilling vessels, thruster assisted moored floating production facilities, shuttle tankers, cable layers, pipe layers, icebreakers and other ice going vessels, supply vessels, and war ships. There is also a significant on-going research and evaluation of using electric propulsion in new vessel designs for existing and new application areas.

The following characteristics summarize the main advantages of electric propulsion in these types of vessels:

- Improved life cycle cost by reduced fuel consumption and maintenance, especially where
there is a large variation in load demand. E.g. for many DP vessels a typically operational profile is equally divided between transit and station keeping/maneuvering operations.

- Reduced vulnerability to single failure in the system and possibility to optimize loading of prime movers (diesel engine or gas turbine).

- Light high/medium speed diesel engines.

- Less space consuming and more flexible utilization of the on-board space increase the payload of the vessel, see Figure 3.1.

- Flexibility in location of thruster devices because the thruster is supplied with electric power through cables, and can be located very independent on the location of the prime mover.

- Improved maneuverability by utilizing azimuthing thrusters or podded propulsion.

- Less propulsion noise and vibrations since rotating shaft lines are shorter, prime movers are running on fixed speed, and using pulling type propellers gives less cavitation due to a more uniform water flow.

These advantages should be weighted up against the present penalties, such as:

- Increased investment costs. However, this is continuously subject for revisions, as the cost tends to decrease with increasing number of units manufactured.

- Additional components (electrical equipment – generators, transformers, drives and motors/machines) between prime mover and propeller increases the transmission losses at full load.

- For newcomers a higher number and new type of equipment requires different operation, manning, and maintenance strategy.

High availability of power, propulsion and thruster installations, as well as safety and automation systems, are key factors in obtaining maximum operation time for the vessel. The safety and automation system required to monitor, protect, and control the power plant, propulsion and thruster system, becomes of increasing importance for a reliable and optimum use of the installation (Blokland and van der Ploeg [33]).

### 3.1.3 Power Flow and Power Efficiency

In any isolated power system, the amount of generated power must be equal to the consumed power including losses. For an electric system consisting of an electric power generation plant, a distribution system, including distribution transformers and a variable speed drive, the power flow can be illustrated in Figure 3.2.

The prime movers, e.g. diesel engines or gas turbines, supply a power to the electric generator shaft. The electric motor, which could be the propulsion motor, is loaded by a power from its connected load. The power lost in the components between the shaft of the diesel engine and the shaft of the electric motor is mechanical and electrical losses which gives heat and temperature increase in equipment and ambient. The electrical efficiency of the system in Figure 3.2 is
Figure 3.2: Power flow in a simplified electric power system.

\[
\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{losses}}},
\]

where \( P_{\text{in}} \) is the generated power, \( P_{\text{out}} \) is the delivered power to the load (thruster), and \( P_{\text{losses}} \) is the losses. For each of the components, the electrical efficiency can be calculated, and typical values at full (rated) power are:

- Generator: \( \eta = 0.95 - 0.97 \).
- Switchboard: \( \eta = 0.999 \).
- Transformer: \( \eta = 0.99 - 0.995 \).
- Frequency converter: \( \eta = 0.98 - 0.99 \).
- Electric motor: \( \eta = 0.95 - 0.97 \).

Hence, the efficiency of a diesel electric system, from diesel engine shaft, to electric propulsion motor shaft, is normally between 0.88 and 0.92 at full load. It must be noted that efficiency is strongly depending on the loading of the system.

Since the additional components between the prime mover and the propeller shaft in a diesel electric propulsion system contributes to a total of approximately 10% losses, the fuel savings potential is not due to the electrical component. One must regard the hydrodynamic efficiency of a speed controlled propeller compared to a fixed speed controllable pitch propeller (CPP), and the fuel efficiency of the prime mover when installed in a diesel electric system with constant speed and high loading, compared to in a mechanical propulsion system with strongly varying load. The differences may be significant, especially on low thrust operations as DP and maneuvering.

Fig. 3.3 shows the fuel efficiency of a typical medium speed diesel engine, and Fig. 3.4 a power vs. thrust comparison of a variable speed and a controllable pitch propeller (CPP).

The hydrodynamic losses will vary significantly dependent on the operational condition for a CPP used in direct driven diesel solutions compared to variable speed fixed pitch propellers (FPP), which normally are used in electrical propulsion. In low load condition it is a rule of thumb that the zero-load hydrodynamic losses for a CPP is about 15%, while it is close to 0 for a speed controlled FPP, see Fig. 3.4. Notice that in most CPP configurations the propeller
speed has to be kept constant on quite high rotations per minute (RPM) even though the thrust demand is zero. For FPP the variable speed drive will allow zero RPM at zero thrust demand. The advantage with CPP is that the propeller pitch ratio will be hydro-dynamically optimized for a wider speed range. A propeller designed for high transit speed, will have reduced efficiency at low speed and vice versa. Hence, the operational profile is of major importance while designing the propulsion system.

The fuel efficiency characteristics of the diesel engine, with maximum fuel efficiency in the load range of 60 to 100% load, strongly contribute to the difference in power consumption for a traditional mechanical propulsion system, and a diesel electric propulsion system. In a power plant for diesel electric propulsion, the power generation will consist of multiple smaller diesel engines, where the number of running aggregates can be selected to have an optimum loading of each engine. The rating of the engines can also be adapted to fit the intended operational profile of the vessel, ensuring that it is possible to find an optimal configuration for most of the operational modes and time.

For a field support vessel, with operational profile as shown in Fig 3.5, it was found that the fuel savings by using diesel electric propulsion was in the range of 700 tons of diesel per year. With a price of approximately 3 NOK per liter, this gives annual savings of in the order of 2.1 million NOK (280 thousand USD). As shown, the savings will strongly be dependent on the operational profile, as shown in Fig 3.6. Here, the operational profile is split in DP/maneuvering and in Transit, showing how an increase portion of DP operations will increase savings, and vice versa.

### 3.1.4 Historical Overview of Electric Propulsion

After the rather experimental applications of battery driven electric propulsion at the end of the 19th century took place in Russia and Germany, the first generation electric propulsion was taken into use in the 1920’s as a result of the strong competition for reducing transatlantic
Figure 3.4: Propeller bollard pull characteristics (example).

Figure 3.5: Operational profile for a field support vessel
crossing times for passenger liners. At that time, the high propulsion power demand could only be achieved by turbo-electric machinery. “S/S Normandie” was one of the most renowned. Steam turbine generators provided electric power that was used to drive the 29MW synchronous electrical motors on each of the four screw shafts. The rotational speed was given by the electrical frequency of the generators. The generators would normally run one propulsion motor each, but there were also possibility for feeding two propulsion motors from each generator for cruising at lower speeds.

With the introduction of high efficient and economically favorable diesel engines in the middle of the 20th century, steam turbine technology and electric propulsion more or less disappeared from merchant marine vessels until the 1980’s.

The development of variable speed electric drives, first by the AC/DC rectifier (Silicon Controlled Rectifier – SCR) in the 1970’s and the AC/AC converters in the early 1980’s enabled the power plant based electric propulsion system, which is typical for the second generation electric propulsion. A fixed voltage and frequency power plant consisting of a number of generator-sets feeding to the same network was supplying the propulsion as well as the hotel and auxiliary power. The propulsion control was done by speed control of the fixed pitch propellers (FPP). These solutions were firstly used in special vessels like survey ships and icebreakers, but also in cruise vessels. “S/S Queen Elizabeth II” was converted to electric propulsion in the mid 1980’s, and later followed the Fantasy and Princess class cruise vessels, several DP vessels, and shuttle tankers. Notice that in direct driven diesel propulsion the thrust is normally controlled by a hydraulic system varying the propeller pitch angle. This is denoted as controllable pitch propellers (CPP).

Podded propulsion was introduced in early 1990’s where the electric motor is installed directly on the fixed pitch propeller shaft in a submerged, rotatable pod. While this concept was originally developed to enhance the performance of icebreakers, it was early found to have additional benefits on hydrodynamic efficiency and maneuverability. After the fist application in a cruise liner, “M/S Elation”, the advantages were so convincing that podded propulsion almost over night became a standard on new cruise liners, Figure 3.7.
Figure 3.7: Cruise vessel “M/S Elation” (lower right) equipped with Azipod propulsion frees up space compared to sisterships (upper left) that can be utilized for other purposes, e.g. grey water treatment.
3.2 Applications

3.2.1 Passenger Vessels – Cruise Ships and Ferries

Passenger vessels, cruise ships, and ferries have very high requirement for on-board comfort regarding noise and vibration. In addition, the reliability and availability is very critical for the safety of the passengers and the vessel. Consequentially, electric propulsion was early evaluated to be beneficial and taken into use.

The list of cruise vessels with electric propulsion is today long and increasing. As the podded propulsion is shown to give significant improvements in maneuverability and fuel costs, with an increase in propulsion efficiency of up to 10% (Kurimo [157]), a large and increasing portion of new-buildings are specified with electrical podded propulsion.

As the environmental concern is increasing, the requirements of reduced emission, spill, and damages on coral reefs by anchoring of the cruise vessels are increasing. Hence, the vessel must maintain its position solely by thrusters controlled by a DP system. This will increase the need for electrical propulsion and podded propulsion in the cruise market even more.

The same restrictions and tax penalties for gas emissions (COx, Nox and Sox) have resulted in that several recent new buildings of ferryboats for fjord and strait crossing have been equipped with electric propulsion. With frequent crossing schedules and quay docking, the improved maneuverability by podded propulsion has significantly reduced the fuel consumption. The propulsion power varies with the size of the vessel, from some few \( \text{MW} \) for smaller ferries up to 30-40\( \text{MW} \) for large cruise liners. The hotel load can be a significant part of the total power installation, for a large cruiseliner typically in order of 10-15 \( \text{MW} \).

Fig. 3.8 shows a schematic overview of the main electrical and automation components in a typical cruise vessel with diesel-electric podded propulsion.

3.2.2 Oil and Gas Exploitation and Exploration: Drilling Units, Production Vessels and Tankers

Still a few years ago extensive oil and gas resources were accessible in shallow waters and could be exploited by fixed drilling and production units. In the North Sea, Gulf of Mexico (GoM), and Brazil as in several other areas, those new resources that remain are found in smaller and/or less available fields in deeper waters. These fields require new cost-effective methods to obtain acceptable economy and profit. Deep-water drilling and floating production have become possible with dynamic positioning or thruster-assisted position mooring. Thruster assisted positioning is applied in the North Sea, Canada, and areas with harsh environment. In Brazil, West Africa, and the planned USGOM (US Gulf of Mexico) installations, the trend has been to rely on mooring without thruster assistance for oil production and dynamic positioning for deepwater drilling.

The thrusters used for station keeping (DP operation) typically also constitutes the main propulsion in transit and maneuvering of the vessel, either all or selected units only.

Typical of these vessels is their large installed thruster power, typically 20-50 \( \text{MW} \). Together with the production, drilling, utilities, and hotel loads, the installed power is typically 25-55 \( \text{MW} \). The typical installation has a common power plant for all these loads, enabling flexibility to operation with high energy-efficiency and high availability. Fig. 3.9 shows a schematic overview of a semi-submersible drilling rig. See Farmer [72], Ådnanes et al. [337] and Ådnanes [335].
Shuttle tankers are used for transport of oil from an offshore facility (platforms, buoys, towers or FPSOs) to a processing or storage terminal onshore. There are numerous different off-loading methods in use. For most of them the shuttle tankers need to maintain a fixed position (station keeping) with high accuracy subject to varying environmental conditions. Therefore, most of the shuttle tankers are equipped with a DP system. Most of the ships have installed electrical tunnel or azimuthing thrusters, whereof some are also having diesel-electrical systems for main propulsion, Hansa-Schiffart [104].

For many applications there is a high degree of redundancy in the propulsion for transit and station keeping. The solutions have normally a redundant power generation and distribution system, with redundant propulsion converters, and a tandem or redundant propulsion motor.

The introduction of podded propulsion may influence the design of the diesel-electric shuttle tankers, since it may be a more cost efficient solution to obtain redundant propulsion with two pod units than with two conventional shaft lines.

### 3.2.3 Field Support Vessels and Construction Vessels

For vessels with dynamic positioning (DP) as the main operating mode, such as diving support vessels, crane ships, and pipe layers (Fig. 3.10 and 3.11), electric propulsion was early taken into use, first with fixed speed CP propellers and later with variable speed thrusters.

The reduction in fuel consumption and environmental emission from diesel electric propulsion compared to conventional mechanical propulsion is significant for vessels with a diversified operational profile. Savings of 30-40% in fuel consumption annually has been reported from ship owners, and with the increased focus on operation costs and environmental impact from the oil
Figure 3.9: Example of electrical system layout for a semi-submersible drilling unit.
industry, has given a large growth in number of field support vessels, first in the North Sea, and later in other geographical areas.

With the rapidly increasing need for high-speed communication system and a global fiber optic cable network, there has been established a large fleet of cable laying vessels with electric propulsion and dynamic positioning.

These vessels will be configured as DP vessels, class 2 or 3 (DnV [54], LRS [174] and ABS [3]), and most will have electric propulsion with a total power demand of 8-30 MW, depending on size and drilling/lifting capability.

3.2.4 Icebreakers and Ice Going Vessels

The dynamic requirements for the frequency converters in propulsion application are low, compared to many other industrial applications. But in ice-going vessels and icebreakers (Figure 3.12 and 3.13), the load variations may be significant and rapid, and this implies that the propulsion system must have high dynamic performance in order to avoid over-loading of components and undesired tripping. Electric propulsion has been used in a majority of new-buildings since the 80’s. The basic configuration can be similar as for service vessels, with a redundant power generation and distribution system, although there will normally not be any DP requirement for the icebreakers.

The installed propulsion power may be in the range of 5-15 MW, depending on ice breaking capability, Cegelec [44], Hill et al. [117] and MacDonald [181].

3.2.5 War Ships

Despite the great interest in the application of electric propulsion to warships, there are quite few conventional surface warships with pure electric propulsion, but more are being projected. For sub-marines, electric propulsion with diesel engine generation and battery storage, fuel cell or nuclear power plant is applied.
Figure 3.11: Some Offshore and Construction Vessels

Figure 3.12: “M/S Botnica”, Icebreaker which serves as a supply vessel in summer season, equipped with Azipod propulsion.
Electric propulsion for war ships does not conceptually differ much from the merchandise vessels, but the solutions may differ since the requirements to availability and redundancy are normally stricter. Also, the ability to withstand shock and provide low noise signatures are prerequisites for electric drive when applied to a warship.

Fig. 3.14 shows the K/V Svalbard, a coast guard vessel in service since 2002 for the Norwegian Navy, equipped with dual Azipod propulsion system, and partially fulfilling military requirements.

### 3.2.6 Research Vessels

Geo technical research vessels, oceanographic vessels, and fishing research vessels have in common very strict underwater noise requirements, typically several decades dB below normal levels for other applications.

This has traditionally been achieved by use of direct propulsion with DC motors, special considerations for filtering and reduction of vibrations and torque variations.

By use of modern frequency converters and filtering techniques, it is likely that AC motors will be feasible for such high demanding applications as well.

### 3.2.7 Trends and New Applications

Electric propulsion is continuously being investigated and evaluated for new applications. LNG and chemical tankers, Ro-Ro vessels, container vessels, fishing vessels are typical examples of large volume markets where electric propulsion yet is not taken into use because of the increased investment costs.

However, only small changes in operation and design criteria, such as increased fuel or emission costs, regulatory restrictions, and equipment cost reduction, may give a tremendous shift in technology application for several of new areas. Figure 3.15 shows the world new build market of ships.
Figure 3.14: "K/V Svalbard", Ice breaking coast guard vessel with podded propulsion for the Norwegian Navy

Figure 3.15: The world new build market of ships. Electric propulsion dominates in the sectors of construction, cable and pipe layers, offshore, icebreakers, and passenger vessels. In other segments, electric propulsion is less in use although there is a significant increase in interest for conceptual studies and designs.
3.3 Overview of Electric Power System

3.3.1 Introduction

The main difference between the marine and a land-based electrical power system is the fact that the marine power system is an isolated system with short distances from the generated power to the consumers, in contrast to what is normal in land-based systems where there can be hundreds of kilometers between the power generation and the load, with long transmission lines and several voltage transformations between them. The amount of installed power in vessels may be high and this gives special challenges for the engineering of such systems. High short circuit levels and forces must be dealt with in a safe manner. The control system in a land-based electrical power system is divided in several separated sub-systems, while in a vessel; there are possibilities for much tighter integration and coordination.

The design of power, propulsion and control systems for a vessel have undergone significant changes and advances over a relatively recent period of time. Because of the rapidly expanding capabilities of computers, microprocessors and communications networks, the integration of systems which were traditionally separate, stand alone systems is now not only feasible, but fast becoming industry standards. The increasing demand for redundant propulsion and DP class 2 and class 3 vessels, requires system redundancy with physical separation. The interconnections of the diverse systems on a vessel have become increasingly complex, making the design, engineering and building of a vessel a more integrated effort.

Fig. 3.16 shows the schematics of the main power installations in a vessel with electric propulsion in a Single Line Diagram (SLD). This chapter describes the main components as they are applied in a marine electric installation:

- Electric Power Generation
- Electric Power Distribution
- Variable Speed
- Propulsion / Thruster units

3.3.2 Electric Power Generation

Prime Mover

The source for power is most often a generator set driven by a combustion engine which is fueled with diesel or heavy fuel oil. Occasionally one can find gas engines, and also gas turbines, steam turbines or combined cycle turbines, especially for higher power levels, in light high-speed vessels, or where gas is a cheap alternative (e.g. waste product in oil production, boil-off in LNG carriers, etc.).

In a diesel-electric propulsion system, the diesel engines are normally medium to high-speed engines, with lower weight and costs than similar rated low speed engines that are used for direct mechanical propulsion. Availability to the power plant is of high concern, and in a diesel electric system with a number of diesel engines in a redundant network; this means high reliability but also sophisticated diagnostics and short repair times.

The combustion engines are continuously being developed for higher efficiency and reduced emissions, and at present, a medium speed diesel engine has a fuel consumption of less than
200g per produced kWh at the optimum operation point as seen in Fig. 3.17a). Even though this is regarded to be a high utilization factor of fuel, it represents only about 40% of the energy in the fuel, the rest of the energy being removed by the exhaust or heat dissipation.

Moreover, the efficiency drops fast as the load becomes lower than 50% of MCR (Main Continuous Rating). At this working condition, the combustion is inefficient, with high NOx and SOx content, and with a high degree of sooting (carbon deposits) which increases the need for maintenance. In a diesel electric system with several diesel engines it is hence an aim to keep the diesel engines loaded at their optimum operating conditions by starting and stopping generator sets dependent on the load, as seen in Fig 3.17b), with an aim to keep the average loading of each running diesel engine closest possible to its optimum load point.

For detailed description of design and functionality of diesel combustion engines, see Mahon [178].

Generators

The majority of new buildings and all commercial vessels have an AC power generation plant with AC distribution. The generators are synchronous machines, with a magnetizing winding on the rotor carrying a DC current, and a three-phase stator winding where the magnetic field from the rotor current induces a three-phase sinusoidal voltage when the rotor is rotated by the prime mover. The frequency $f \,[Hz]$ of the induced voltages is proportional to the rotational speed $n \,[rpm]$ and the pole number $p$ in the synchronous machine

$$f = \frac{p \, n}{2 \, 60}. \tag{3.2}$$
Figure 3.17: a) (left) Example fuel consumption for a medium speed diesel engine. b) (right) Total efficiency from engine to propeller shaft, in a single machine direct mechanical propulsion system and a four machine diesel electric propulsion system.

A two-pole generator will give 60 Hz at 3600 rpm, a four-pole at 1800 rpm, and a six-pole at 1200 rpm, etc. 50 Hz is obtained at 3000 rpm, 1500 rpm, 1000 rpm for two-, four-, and six-pole machines. A large medium speed engine will normally work at 720 rpm for 60 Hz network (10 pole generator) or 750 rpm for 50 Hz networks (8 pole generator).

The DC current was earlier transferred to the magnetizing windings on the rotor by brushes and slip rings. Modern generators are equipped with brushless excitation for reduced maintenance and downtime. The brush-less excitation machine is an inverse synchronous machine with DC magnetization of the stator and rotating three-phase windings and a rotating diode rectifier. The rectified current is then feeding the magnetization windings.

The excitation is controlled by an automatic voltage regulator (AVR), which senses the terminal voltage of the generator and compares it with a reference value. Simplified, the controller has PID characteristics, with stationary limited integration effect that gives a voltage drop depending on the load of the generator. The voltage drop ensures equal distribution of reactive power in parallel-connected generators. According to most applicable regulations, the stationary voltage variation on the generator terminals shall not exceed ±2.5% of nominal voltage. Also, the largest transient load variation shall not give voltage variation exceeding -15% or +20% of the nominal voltage unless other has been specified and accounted for in the overall system design. In order to obtain this transient requirement, the AVR is normally also equipped with a feed-forward control function based on measuring the stator current.

In addition to the magnetizing winding, the rotor is also equipped with a damper winding which consists of axial copper bars threaded through the outer periphery of the rotor poles, and short circuited by a copper ring in both ends. The main purpose of this winding is to introduce an electromagnetic damping to the stator and rotor dynamics. A synchronous machine without damper winding is inherently without damping and would give large oscillations in frequency and load sharing for any variation in the load.

In section 3.5, stationary, transient and sub-transient models will be introduced. Simplified
one could say that the flux linkages in the damper winding, which are “trapped” and resist changes due to being short-circuited, characterize the sub-transient interval. This is observed as an apparent lower inductance in the generator, which gives a stiffer electric performance during quick load variations, and helps to reduce transient voltage variations and the voltage variations due to harmonic distortion in load currents. This effect is only contributing for dynamic variations faster than characterized by the sub-transient time constant such as the first period of motor start transients and transformer inrush, and for harmonic distorted load currents.

Often, the generators are connected to a propulsion engine’s shaft, i.e. a shaft generator. The shaft generators are in some applications made for two-directional power flow, which means that it can be run as motor. This principle may be called a PTI-PTO concept (Power take-in – Power take out). Shaft generators have the disadvantage of forcing the main propeller to work at fixed speed if the generator output shall have constant frequency. This will reduce the efficiency of the propeller in low load applications. Static converters may be installed to keep fixed frequency for variable speed.

### 3.3.3 Electric Power Distribution

#### Switchboards

The main (or generator) switchboards are usually distributed or split in two, three, or four sections, in order to obtain the redundancy requirements of the vessel. According to rules and regulations for electric propulsion, one shall tolerate the consequences of one section failing, e.g. due to a short circuit. For strictest redundancy requirements, one shall also tolerate failure due to fire or flooding, meaning that water and fireproof dividers must be used to segregate the sections.

In a two-split configuration, with equally shared generator capacity and load on both sides, the maximum single failure scenario will hence be to loose 50% of generator capacity and loads. In order to avoid high installation costs, the system will often be split in three of four, which reduces the required additional installations. Also, change-over switches which ensures that a generator or a load can be connected to two switchboard sections will have similar cost reducing effects, e.g. for azimuth thruster in Fig.3.16.

In propulsion mode, the switchboards are normally connected together, which gives the best flexibility in configuration of the power generation plant. The load transients are distributed on a large number of diesel-generators, and the most optimal number of units can be connected to the network.

Another possibility is to sail with independent switchboard parts supplying two or more independent networks. In this case the ship is often assumed to be virtually blackout proof, which could be attractive in congested waters. In this operating mode one network including its connected propulsion units is lost if one switchboard section fails, the other, however, remaining operable. In practice, there are also other considerations to be made in order to obtain such independence, especially all auxiliaries, such as lubrication, cooling, and ventilation must be made independent. Also, loss of propulsion or station keeping power on one part of the system, will through control systems also have impact on the remaining parts, as the total power or thrust tends to be kept the same, e.g. for dynamic positioning.

The normal operation in DP vessels, in particular for class 3 operations, is to split the network in order to be tolerant to failure of one section. However, rules and regulations now
allows for operation with closed tie breakers, if the protection circuits are designed to detect and isolate faulty parts without tripping the healthy parts. The NMD rules (Norwegian Maritime Directorate) has one of the more stricter practicing of these rules and will normally not accept connected networks in class 3 operations.

As the installed power increases, the normal load currents and the short circuit currents will increase. With the physical limitations on handling the thermal and mechanical stresses in bus bars and the switching capacity of the switchgear, it will be advantageous or necessary to increase the system voltage and hence reduce the current levels. Medium voltage has become a necessity to handle the increasing power demand in many applications. Using the IEC voltage levels the following alternatives are most common selected for the main distribution system, with application guidelines from NORSOK [207]:

- **11kV**: Medium voltage generation and distribution. Should be used when total installed generator capacity exceeds 20MW. Should be used for motors from 400kW and above.

- **6.6kV**: Medium voltage generation and distribution. Should be used when total installed generator capacity is between 4 – 20MW. Should be used for motors from 300kW and above.

- **690V**: Low voltage generation and distribution. Should be used when total installed generator capacity is below 4MW. Should be used for consumers below 400kW and as primary voltage for converters for drilling motors.

- **For utility distribution lower voltage is used, e.g. 400/230V.**

In US, or where the ANSI standard applies, several additional voltage levels are recognized, such as in IEEE [125]: 120V, 208V, 230V, 240V, 380V, 450V, 480V, 600V, 690V, 2400V, 3300V, 4160V, 6600V, 11000V, and 13800V. The 3300V is also a commonly used system voltage in IEC applications, even though not recognized in NORSOK [207].

Since the load current and fault current determine the limitation of the equipment, the actual power limits for each system voltage may deviate from these recommendations. This particularly applies to systems where a major part of the load is converter loads and does not contribute to short circuit power. Since these do not contribute to short circuit currents in the distribution system, it often allows increasing the power limits for the different voltage levels.

Also, the selection of system voltage level can be influenced by other criteria, such as availability of equipment. A lot of ship equipment is available only in 440V, which means that it might be difficult to avoid this voltage level in ship applications.

Safety is an issue of concern when yards and ship owners changes from low to higher voltages, often leading to a misunderstanding effort to keep voltages as low as possible. In the context of safety, it should be regarded that medium voltage switchboards is designed to prevent personnel to get contact with conductors, even in maintenance of the switchgears. The normal and fault currents are similarly smaller, giving less forces on the conductors and cables during e.g. short circuit. Although short circuits inside the switchboards are extremely rare, arc-proof design (IEC 298-3) is available and will prevent person injury and limit the equipment damages if worst case should occur.

Circuit breakers are used for connecting and disconnecting generator or load units to the switchboards, or different parts of the switchboards together. Various circuit breaker technologies are applied. Air insulated units are the traditional solution, but today rarely applied except
at low voltage levels. In the commonly used SF6 and vacuum breaker technologies, the current interruption takes place in an enclosed chamber, where the first one is filled with SF6 gas, which has higher insulation strength than air, and the vacuum breaker is evacuated by air. These designs give compact and long term reliable solutions for medium voltages. One should consider that vacuum breakers may chop the current and can cause overvoltage spikes when breaking an inductive loads with high di/dt that may require installation of overvoltage limiters.

For smaller powers, fused contactors are a cost and space beneficial alternative to the circuit breaker, and are available in air (low voltage), SF6 or vacuum insulated types. The problem with switching spikes is less with fused contactors since current interruption is softer (lower di/dt).

**Transformers**

The purpose of the transformer is to isolate the different parts of the electric power distribution system into several partitions, normally in order to obtain different voltage levels and sometimes also for phase shift. Phase shifting transformers can be used to feed frequency converters, e.g. for variable speed propulsion drives, in order to reduce the injection of distorted currents into the electric power network by canceling the most dominant harmonic currents. This reduces the voltage distortion for generators and other consumers. The transformers also have a damping effect of high frequency conductor emitted noise, especially if the transformer is equipped with a grounded copper shield between primary and secondary windings.

There are numerous different transformer designs in use, and the most common types are; air insulated dry type, resin insulated (cast or wound), or oil/fluid insulated. Regulations, ambient conditions, and user’s, yard’s, or supplier’s preferences govern the selection of type, material, and design of the transformer.

Physically, the transformer is normally built as three-phase units, with three-phase primary coils and three-phase secondary coils around a common magnetic core. The magnetic iron core constitutes a closed path for magnetic flux, normally with three vertical legs and two horizontal yokes; one in bottom and one at top. The inner winding constitutes the low voltage or secondary windings, and the outer is the primary or high voltage winding. The ratio of primary to secondary windings gives the transformation ratio. The coils may be connected as a Y-connection or Δ-connection (also called D-connection). The connection may be different on primary and secondary sides, and in such transformers, not only the voltage amplitude will be converted, but there will also be introduced a phase shift between the primary and secondary voltages.

A transformer with Δ-connected primary and Y-connected secondary is called a Dy type transformer. The first and capital letter describes the primary winding, and the second and small letter describe the secondary winding. The letter n is used to described if the common point in a Y-connection is grounded, e.g. Dyn or Yynn.

Transformers may be designed according to IEC standards. For converter transformers, it is essential that the design accounts for the additional thermal losses due to the high content of harmonic currents. IEC also gives design rules and guidelines for such applications.
3.3.4 Motor Drives for Propulsion and Thrusters

Introduction

The electrical motor is the most commonly used device for conversion from electrical to mechanical power and is used for electric propulsion, thrusters for propulsion or station keeping, and other on-board loads such as winches, pumps, fans, etc. Typically, 80-90% of the loads in ship installations will be some electrical motors.

In this chapter, a brief overview of different motors and their applications in ship installations is given and for more detailed description of design, performance, and characteristics, references to other books are made.

The electrical motors in use are:

- **DC motors**: The DC motor must be fed from a DC supply, and since the power generation and distribution system normally is a three-phase system, this means that a DC motor must be fed from a thyristor rectifier. This gives also a speed control of the motor. For detailed description of the various construction of the DC motor, see Fitzgerald et al. [77].

- **Asynchronous (induction) motors**: The asynchronous or induction motor is the workhorse of the industry. Its rugged and simple design ensures in most cases a long lifetime with a minimum of breakdown and maintenance. The asynchronous motor is used in any applications, either as a constant speed motor directly connected to the network, or as a variable speed motor fed from a static frequency converter. Fitzgerald et al. [77] gives a good explanation on design and operating performance of asynchronous motors.

- **Synchronous motors**: The synchronous machine is normally not used as a motor in ship applications, with exception of large propulsion drives, typically >5 MW directly connected to propeller shaft, or >8-10 MW with a gear connection. In power range smaller than this, the asynchronous motor is normally cost-competitive. The design of a synchronous motor is similar to that of an synchronous generator. It is normally not used without a frequency converter supply for variable speed control in ship applications. See Fitzgerald et al. [77] for design and operating performance.

- **Permanent magnet synchronous motors**: Permanent magnet synchronous motors are used in industrial drives for some few kW drives, also for direct on-line applications. In recent years, it has been introduced also for large power applications; in several MW propulsion drives, firstly in navy applications but now also in podded propulsion applications. The benefit of this design is high efficiency with compact design, making it especially interesting for podded propulsion where the dimensions should be as small as possible, and direct water cooling would eliminate the need for air cooling of the pod motor and simplify the construction and installation work. See Ådnanes [334] for description of design and performance.

- **Other motors**: A range of other motors is used in commercial or experimental applications. Few of them have gained a high market share, and especially not in marine applications. It might in future be seen some new concepts for variable speed drives, based on motor designs with higher efficiency, small dimensions, or specially designed for certain applications. They will most likely though be based on the principles described above or derivatives of these.
The constant speed, Direct-on-Line motor

An electric motor can be directly connected to the network, and such direct-on-line (DOL) motors are normally three-phase asynchronous, or induction motors. The asynchronous motor has a rugged and simple design, where the three-phase stator windings are similar to a generator stator winding. The rotor is cylindrical, with a laminated iron core and a short circuited winding similar to the damper winding in a synchronous machine. At no-load, the voltages imposed to the stator winding will set up a magnetic field in the motor, which crosses the air gap and rotates with a speed given by the frequency of the imposed voltages, called synchronous frequency, $f_s$. The synchronous speed $n_s$ in rpm is hence

\[ n_s = \frac{f_s \times 60}{\pi}. \]  \hspace{1cm} (3.3)

As the shaft gets loaded, the rotor speed will decrease, and there will be induced currents in the rotor winding since they are rotating relatively to the synchronous rotating magnetic field from the stator windings. One defines the slip, $s$, as the relative lag of motor speed to the synchronous speed $n_s$

\[ s = \frac{n_s - n}{n_s}. \]  \hspace{1cm} (3.4)

Hence the slip varies from 0 (no-load) to 1 (blocked rotor). The slip at rated load is normally below 0.05 (5%) for most motor designs, and even lower (2−3%) for large motors.

>From the electrical model of the asynchronous motor, a mathematical formula for the rotor and stator current, shaft torque, and power can be developed as a function of slip. A complicating factor is that the parameters, especially the rotor parameters, are very dependent on the slip, i.e. frequency of the rotor currents, and these frequency dependencies must be regarded in order to obtain accurate results.

Figure 3.18 shows the stator currents and shaft torque for an asynchronous motor which is connected to a fixed frequency and stiff network, as function of rotor speed or slip. It also indicates the load curve for a typical CPP thruster application at zero pitch and full pitch. Start-up of a thruster motor should always be done with zero pitch in order to make sure that sufficient torque margin is available to secure start-up, and to minimize the starting time.

Under stationary conditions, the motor speed is close to synchronous speed, and the induced rotor currents are nearly proportional to the slip, and also the shaft torque. From the electrical model in Figure C.14, one can then derive a simplified expression of the resulting rms stator current, $I_s$

\[ I_s = \sqrt{I_m^2 + I_r^2} = \sqrt{I_{mN}^2 + I_{rN}^2 \frac{T}{T_N}}. \]  \hspace{1cm} (3.5)

where $I_m$ is the magnetizing current flowing through the magnetizing inductance $L_m$, neglecting magnetizing losses in $R_m$. $I_r$ is the rotor current referred to stator side, and $T$ is the torque. A subscript $N$ annotates quantities under nominal (rated) conditions, and simplified one may express, if neglecting effects of leakage inductance in rotor and stator

\[ I_{mN} = I_{sN} \cos \phi_N, \]  \hspace{1cm} (3.6)

\[ I_{rN} = I_{sN} \sin \phi_N. \]  \hspace{1cm} (3.7)

When the slip approaches the peak torque and higher, the assumptions are not valid any longer, since the effects from neglecting leakage inductances becomes considerable, and the load stator
current will typically follow a characteristics as shown in Figure 3.18, typically with a quite flat current amplitude of approximately five times nominal current (locked rotor current).

Due to the high starting current of asynchronous machines, it will often be necessary to install devices for soft starting. Soft starters typically reduce the locked rotor currents from 5 times to 2-3 times nominal current, and thereby also reduce the voltage drop. Soft starters must always be adapted to the load characteristics, as their principle is based on reducing the motor voltage at start-up, and hence reducing the torque capability of the motor. The most commonly used are:

- Star-delta \((Y - \Delta)\) coupling
- Autotransformer
- Semiconductor (thyristor) soft starter

**Variable speed motor drives and control strategies**

The direct on line motor will rotate with a speed directly determined by the network frequency. For propulsion, thrusters, pumps, winches, etc., there might be significant savings in power or fuel consumption by reducing the no-load dependent losses in operations. Also, controllability of the driven load will be greatly enhanced by controlling the speed of the motor. The penalty is primarily economically, by introducing additional investment costs, and also components that require maintenance. The additional investment should paid back by reduced operating costs or increased earnings if the investment shall be justified. With an energy cost of approximately 1NOK per kWh for generated power onboard a vessel, one will briefly save 8 – 9MNOK annually for a 1MW average power reduction.
The most commonly used motor drives are:

- Voltage source inverter (VSI) type converters for AC motors, normally asynchronous motors
- Current source inverter type (CSI) converters for AC motors, normally synchronous motors
- Cycloconverters (Cyclo) for AC motors, normally for synchronous motors
- DC converters, or SCR (Silicon Controlled Rectifier) for DC motors

In ships, the most used variable speed drives uses AC motors. Most drives, except the cycloconverter, will consist of a rectifier which rectifies the line voltage, and an inverter, which generates the variable frequency and variable voltage source for the motor. More detailed description of these concepts will follow in a later section.

A motor controller contains the speed control, and the control of motor currents by controlling the switching elements of the rectifier and/or inverter. An interface to an overriding control system, vessel management system, maneuvering control, or dynamic position control is normally required. The motor controller acquires measurement signals and feedback signals from sensors in the drive, and motor. Typically motor currents, motor speed, and in some cases temperatures and voltages are measured.

The semiconductor components of the power electronic circuits are either uncontrollable (diodes) or controllable (thyristors, IGBTs, IGCTs). Fig 3.20 shows a low voltage IGBT module, containing all switching elements for a 690V inverter module, and Fig. 3.21 a discrete medium voltage IGCT used in 3300V inverters.

Power electronics is regarded as a separate field of science, and for further studies, Bose [36] and Mohan et al. [191] can be recommended.
Figure 3.20: An IGBT with the encapsulation removed. This consists of several integrated components in one module.

Figure 3.21: IGCT “hockey puck” opened, showing its building components. The silicon wafer is on top (middle).
A motor can, if designed for it, run in both direction, with either a driving (motoring) or breaking shaft torque. In order to categorize what conditions the motor drive is designed for, the quadrant terms are often applied. The quadrants refer the four quadrants of a speed-torque diagram, as shown in Figure 3.22. The motor is motoring, i.e. running the load with power input to the load shaft in quadrants I and III. Oppositely, the motor is breaking, i.e. mechanical power is transferred from the load to the drive, when operating in quadrants II and IV.

The motor drive normally comprises a speed control function, and the output from this control function can be interpreted as a torque command or reference which is the input to the motor control algorithms. These algorithms use a more or less advanced motor model to control the motor currents and voltages by turning on or off the switching elements of the rectifier (if controllable) and the inverter.

In principle, the controllers will normally have a control block diagram as shown in Figure 3.23. Torque control is achieved by removing the speed control loop, and give torque reference as a direct input to the motor drive as shown with dashed lines in Figure 3.24. Motor speed is normally measured, but new motor controllers are equipped with motor speed estimator which eliminate the need for a dedicated speed sensor for most ship applications.

For most practical reasons, the speed control loop of a motor drive can be regarded as a PI (or PID) controlled closed loop with an inner closed torque control loop, which for control purposes be regarded as a first order time lag. For simulations and synthesis of overriding control loops, the simplified block diagram in Fig 3.24 should be applicable.
Figure 3.23: A generic and typical control block diagram for a motor drive controller.

Figure 3.24: Simplified block diagram for simulations and synthesis of overriding control loops.
3.3.5 Propulsion Units

Introduction

This section presents the most commonly used principles of propulsion units in vessels with electric propulsion. The overview is not complete, since there are also other alternatives, e.g. water jets, also in use, however only for special and limited applications.

Shaft propulsion

In a diesel-electric power and propulsion system with shaft propeller, the propellers are normally driven by variable-speed electric motors. The horizontal motors may be directly connected to the shaft, which results in a simple and mechanically robust solution, or via a gear coupling, which allows for increased rotational speed of the motor and results in a much more compact motor. The disadvantage is increased mechanical complexity and increased mechanical power losses.

In diesel electric vessels, shaft lines are used in applications typically where the propulsion power is higher than available for azimuthing thrusters, or where the ability to produce transverse thrust, e.g. in station keeping and maneuvering is not needed – or can be produced cheaper by tunnel thrusters. Typically this applies for shuttle tankers, research vessels, larger anchor handler vessels, cable layers, etc.

The shaft line propulsion will always be combined with rudders, one rudder per propeller. By use of high-lift rudders, shaft propellers may also be used to provide a certain degree of transverse thrust. If additional transversal thrust is needed for maneuvering or station keeping, there will normally be required to install additional tunnel thrusters also in the aft of the vessel.

The propeller is normally speed controlled FPP (Fixed Pitch Propeller) type, which gives a simple and robust propeller design. In some applications, the propeller may be CPP (Controllable Pitch Propeller) type, even if it is speed controlled. To a certain degree, speed and pitch can be optimized for higher efficiency, and faster response than with only one control parameter. These benefits normally do not justify the additional investments in order to obtain combined speed and pitch control.

Fig 3.25 shows some typical drive configurations for shaft line propulsion system. These can be installed in single shaft propeller designs, or dual shaft designs.

Azimuth thrusters

Azimuth thrusters are thrusters that can be rotated in order to produce thrust in any direction. The thrust is controlled either by constant speed and CPP design, variable speed FPP design, or in rare cases with a combination of speed and pitch control. Variable-speed FPP designs has a significantly simpler mechanical underwater construction with reduced low-thrust losses compared to constant speed, CPP propellers.

In vessels with strict limitation of in-board height of the thruster room, the electric motor will normally be horizontal, and the azimuthing thruster will then consist of a Z-type gear transmission. Due to a simpler construction with less power transmission losses, vertically mounted motors and L-shaped gear transmission will normally be selected when the height in the thruster room allows for it.

A limitation of azimuth thrusters is their limited ability for producing thrust at negative pitch or RPM, because they are designed and optimized for unidirectional thrust. If they have
a certain degree of negative thrust capability this should be utilized in order to maintain dynamic thrust capacity without performing continuous azimuth rotation.

The conventional azimuth thruster was earlier used for station keeping and maneuvering, but has recently also been taken in use as the main propulsion device in vessels with electric propulsion. In order to improve the hydrodynamics and steering capability that is required for propulsion, the shape of the thruster has been adapted, such as the “mechanical pod”. This is an azimuthing thruster, which is powered from an in-board, typically a horizontal motor, and the mechanical power is then transferred to the propeller with a Z-shaped gear. The underwater shape is optimized for low hydrodynamic resistance at higher ship velocity, for higher propulsion efficiency.

Some vendors can supply thruster devices with dual propeller, either on the same shaft, or with contra-rotating propellers. Contra rotating propeller increases the hydrodynamic efficiency by utilizing the rotational energy of the jet stream from one propeller, to create thrust from the other that rotates the opposite direction. Conventional azimuth thrusters are at present (2002) in use with power ratings up to 6-7 MW.

**Podded propulsion**

Like the conventional azimuth thruster, the podded propulsion unit is freely rotate-able and may produce thrust any direction. The main difference is the integration of the electrical motor directly to the propeller shaft, inside a sealed pod unit that is submergent under the vessel hull.

The high power pod schematically drawn in Fig 3.26 shows the variable-speed electric motor, which is located in the sealed and compact pod. The fixed-pitch propeller is mounted directly on the motor shaft. Since a mechanical gear is avoided, the transmission efficiency is higher than in an azimuth thruster. The electrical power is transferred to the motor via flexible cabling or slip rings for 360-degree operation. Since the propeller pitch is fixed and there is no gear transmission, the mechanical construction has a lower mechanical complexity.
The pod can be designed for pushing or pulling operation. Especially the pulling type pod gives the propeller a near optimum and uniform wake field, which increases the hydrodynamic efficiency of the propeller and reduces the risk for cavitation, and hence give reduced noise and vibrations. A podded unit can rotate in both forward and aft directions if the thrust bearings are designed for it. The propeller is normally optimized for one main thrust direction, giving some reduced negative thrust capacity, but without the mechanical limitations of the mechanical thruster.

Podded propulsion units have been in operation in a decade in cruise vessels, icebreakers, service vessels and tankers. Recent new-built field support vessels, Fig 3.10, and semi-submersible drilling units are now also utilizing podded propulsion as station keeping/transit propulsion thrusters. The system is today available in power ranges from approximately 1 MW up to at least 25 MW. The larger units provide access into the pod for visual inspection.

3.3.6 Trends and New Concepts

Electric power generation

Electric power generation based on rotating prime movers and electric generators, is a mature technology, but there is continuously being evaluated alternatives to the traditional synchronous generator. The fuel cell technology is an area of great interest and research effort, mainly in the automotive industries.

A fuel cell is an electrochemical device that combines a fuel, e.g. hydrogen, and oxygen from the air to produce electricity, heat and water. Fuel cells operate without combustion, see Fig. 3.27; hence, a hydrogen fuel cell is virtually pollution free. Since the fuel is converted directly to electricity, a fuel cell can operate at much higher efficiencies than internal combustion engines,
extracting more electricity from the same amount of fuel. The fuel cell itself has no moving parts - making it a quiet and reliable source of power. The fuel cell is composed of an anode (a negative electrode that repels electrons), an electrolyte membrane in the center, and a cathode (a positive electrode that attracts electrons).

There is a range of various concepts under evaluation, such as; Phosphoric Acid, Proton Exchange Membrane or Solid Polymer, Molten Carbonate, Solid Oxide, Alkaline, Direct Methanol Fuel Cells, Zinc Air Fuel Cells, and Protonic Ceramic Fuel Cells.

Common for any of these alternatives is that the technology at present has a high production and maintenance cost – with high energy price. The dynamic capability should still be improved, and with the present low power density of the unit the use is in practice limited to smaller power ratings. There are reported some experimental vessel designs using fuel cell power generation, and is commonly assumed and expected that fuel cell will become commercially feasible within some time, but not in the very close future.

The fuel cell generates $\Delta \mathcal{E}$ voltage and one could imagine two alternatives for distribution:

- $\Delta \mathcal{E}$ distribution and $\mathcal{E}$ load – most of the converter principles for propulsion drives are based on rectifying the $\mathcal{A} \mathcal{C}$ supply before converting to variable frequency. The problem is mainly related to switching and protection of the distribution system, since interruption of high $\mathcal{D} \mathcal{C}$ currents is difficult. The conversion to different voltage levels and supply of consumers that need $\mathcal{A} \mathcal{C}$ supply will also become expensive. $\mathcal{D} \mathcal{C}$ distribution has mainly been evaluated, and to some extent been used, in military applications.

- $\mathcal{D} \mathcal{C}$-$\mathcal{A} \mathcal{C}$ conversion of the power. With this solution, the problems with voltage distribution and supply will be reduced, but for higher power levels, the solution is yet expensive.

$\mathcal{D} \mathcal{C}$ distribution could also enable the use of compact and lightweight high-speed permanent magnet exited generators. As the power density of a rotating machine increase by its nominal rotational speed, there would be significant savings in weight and volume if speed could be
increased from typically 500-900 RPM to 15000 RPM. The disadvantage will be a high frequency output with uncontrolled and varying voltage that would require large power conversion installations.

The fuel cell and power electronics developments open for large shifts in applied technology. The electric power generation, distribution, and propulsion technology, which has been applied until recently are lagging state-of-the-art industrial and automotive research and application, but these technologies will make progress step by step just similar to development of other new technologies.

**Electric power distribution**

It is not likely that there will be a dramatic shift in technology in electric power distribution for AC systems. A gradually introduction of new protection relays, programmable and with field-bus communication is already started and will continue. This means that as flexibility will increase, the total project costs will be reduced.

There is research and development of power-formers, static power converters that transforms voltages by use of power electronics. A high frequency transformer can give galvanic insulation, and the benefits will be reduced dimensions and weights, and elimination of transformer inrush problems. It is yet work to be done before power-formers will be applied in commercial vessels, but they will be essential components in order to enable future development of DC distribution systems.

**Propulsion**

The podded propulsion system is the largest step forward as propulsion unit in the recent years. The concept is not yet fully developed for all market and power applications, and there will be a continuous development to be seen. This may have significant impact on vessel design and allow for total new ship concepts for many areas.

In the CRP concept (Contra Rotating Pod), the podded propulsion is combined with a traditional shaft line driven propeller is shown in Fig. 3.14. The Pod is azimuthing and controlled with a variable speed electrical motor, and the shaft line propeller can be either speed controlled with electrical motor, or a conventional pitch controlled direct diesel driven propeller. The CRP concept has shown to give a large improvement in propulsion efficiency, as well as increased redundancy and propulsion power for a several ship types.

**3.4 Power and Propulsion Control**

**3.4.1 Introduction - Control Hierarchy**

Fig. 3.29 may illustrate a modern, integrated control, monitoring, and protection system of a power and propulsion plant system, and the implemented functionality can be described in control hierarchy of Fig. 3.30.

The user interface with presentation of status and measurements, input of operator commands, alarm handling, etc., is often implemented in operator stations, with a graphical user interface, push buttons etc. Operator stations are placed on bridge, engine control rooms, etc.

The system level controllers are implemented in control stations or PLCs. They can be centralized or distributed computers, depending on design philosophy for the vessel. In these
Figure 3.28: The Contra Rotating Pod (CRP) concept.

Figure 3.29: Integrated control system for a vessel, generic configuration.
one will find the energy management functions, such as power management, blackout prevention functions, start-up and reconfiguration sequence control.

Due to the need for separate testing, response time requirements, and vendor’s responsibility, there will be a need for low-level fast-response control, monitoring, and protection of devices. Here are the fast control functions and most basic safety functions implemented. These are linked to the system control level by hard-wired or field-bus signal interface.

3.4.2 User Interface

The user interface, represented by the bridge consoles and monitors, are the crew’s contact for monitoring and control of the power and propulsion installations.

It has become more common to utilize GUI (Graphical User Interface) in ship applications, where light bulbs and push buttons were common few years ago. This allows for much more flexible and cost-efficient solutions, but also introduce a major challenge for vendors and users to specify and design a good user interface which combines safety issues with user friendliness, logical design and easy access to essential and desired information.

3.4.3 High Level Control Functionality

Power management – Energy Management

In a system of electrical power installations, vessel and process automation system, and positioning system, the various parts of the automation system controls their parts of the power system, e.g. the dynamic positioning system controls the thruster drives, the off-loading control system use cargo pump drives, the process control system interacts with compressors and cooling/heating systems etc. The interconnecting point for all installed power equipment is the power distribution system. By starting and inrush transients, load variations, and network disturbances from harmonic effects the load and generators are interacting and influencing each other. Optimum operation and control of the power system is essential for safe operation with
a minimum of fuel consumption. As it is the energy control system (energy, and power management system), which monitors and has the overall control functionality of the power system, it will be the integrating element in a totally integrated power, automation and positioning system.

The purpose of the Power Management System (PMS) is to ensure that there is sufficient available power for the actual operating condition. This is obtained by monitoring the load and status of the generator sets and the power system. If the available power becomes too small, either due to increased load or fault in a running generator set, the PMS will automatically start the next generator set in the start sequence. A power management system can also have extended functionality by monitoring and control of the energy flow in a way that utilizes the installed and running equipment with optimum fuel efficiency. Such systems can be called Energy Management System (EMS).

For PMS and EMS systems, the main functions can be grouped in:

• Power generation management: Overall control with frequency and voltage monitoring with active and passive load sharing monitoring and possibly control, and load dependent start and stop of generator sets. Since control logic and interlocking functions are a significant part of the power system switchboard design, the functionally of these systems must be coordinated.

• Load management: Load power monitoring and coordinator of power limitation functions in other systems, load shedding and start interlock of heavy consumers based on available power monitoring.

• Distribution management: Configuration and sequence control of reconfiguring the power distribution system. The distribution system should be configured to fit the requirements in the actual operational mode for the vessel.

The new generation production vessels and also drill ships/rigs have a complex power system configuration with advanced protection and relaying philosophies. There are close connections between the functional design and performance of the energy control system (power management system) and the power protection system functions. It is a challenge for involved parties to obtain an optimal and functional solution with several suppliers involved and a yard being responsible for all coordination.

Blackout of the power generating system is the most severe fault that can happen in an electric propulsion system. Various mechanisms to avoid blackout are linked to the power management system, such as the auto start/stop functions, reduction of propulsion and other loads, or shedding of non-critical loads. Figure 3.31 illustrates the coordination diagram for a typical installation. Normally, the available power will be controlled within the boundaries for auto start/stop, but if a sudden increase in load, or tripping of a generator set should occur, the available power can be reduced. By monitoring load balance and/or network frequency, the load reduction and load shedding functions will then be activated to reduce the loading and safeguard the power generation until a new generator set is started and connected.

Should a blackout occur, and it does unfortunately happen from time to time, there will normally be required to have a system for sequence control of start-up and reconfiguration of the power system. This is implemented at the system control level, and includes sequences for starting and synchronizing generator sets and loads. There will normally also be a set of predefined operation modes, e.g. transit mode, station keeping mode, maneuvering mode, etc. with automatic sequence control for power system reconfiguration.
Vessel management

In the Vessel Management term, it is common to include manual, automatic, and semi-automatic control of the vessels auxiliaries and helping systems, such as valves, HVAC (Heat, Ventilation, and Air Conditioning) system, ballast control, cargo control, etc. Also the alarm systems, watch call system, and safety systems, may be integrated in the Vessel Management system.

Most of the functions are normally monitored and possible to control from the bridge, or locally from control stations close to the system or engine control room.

Propulsion control and dynamic positioning

The propulsion control systems normally consist of

- Manual Thruster Control (MTC) system providing individual control of the thrusters and propellers.
- Autosail or auto pilot system performing automatic course keeping and course changing during transit operations, often in combination with tracking functionality.

And if the vessel is intended for use in station keeping operations, either

- Dynamic Positioning (Dynpos) System providing manually or automatically positioning by means of proper action of the thruster system, or
- Thruster assisted position mooring (Posmoor ATA) system providing manually (Posmoor TA) or automatically thruster assistance (Posmoor ATA) for position and heading control of anchored vessels.

It is not the intention of this system to describe these control systems in detail, for further studies, Fossen [78] can be recommended.
It should, however, be emphasized, that the propulsion and station keeping control functions are in most cases very critical for the safety of operations of the vessel. The need for careful consideration of their design is obvious, but one should never underestimate the need for matching and testing these control functions together with the power / energy management system, and low level controllers in switchboards, and propulsion drives.

3.4.4 Low Level Control Functionality

One may divide the low-level controllers in protection and control functions. The protection devices shall monitor the units for faults and from exceeding design constraints.

The low-level controllers are dedicated controllers for the purpose, often integrated with the equipment.

Engine protection and governor

The engine protection devices prevent and shut down the engine at over-speed, excessive temperatures, loss of lubrication, etc. The engine protection is usually delivered as an integral part of the engine, by the engine vendor, or partially integrated with vessel management system.

The governor controls the generated frequency by commanding fuel input to the prime mover, Fig. 3.32. It may be a so-called “speed droop” type, which implies that the steady-state frequency will drop proportionally with the active load (kW). The speed droop mode is a simple and robust method for obtaining load sharing between parallel-connected generators. However the load dependent frequency variations may cause difficulties in synchronizing generators or different bus sections. The frequency variation may also be undesired from the operation of loads. The isochronous governor contains a regulator with an integral effect and keeps the frequency equals to the set point. A signal, either hardwired or by high-speed bus-communication, between the governors, ensures a proper load sharing between the prime movers. Most governors applied in electric propulsion plants have both control modes available.

Automatic voltage regulator

The Automatic Voltage Regulator, AVR (see Fig. 3.33), controls the voltage by commanding magnetizing current to the field winding of the generator. As the governor, the voltage regulator can be controlled in droop mode, meaning that voltage varies a few percents (±2.5%) with the load. This gives a robust and simple sharing of reactive load (kVAr), which is a prerequisite for equal loading of the generators and the voltage variations normally gives no impact on synchronization and functionality. In some applications, where the voltage variations are not accepted, an integrating controller will adjust the voltage set point in order to keep the output voltage constant, but the droop mode control is normally preferred.

Protection relays

The generator protection prevents and disconnects the generator from the switchboard at excessive currents in the winding, short circuits, ground faults, faulty synchronization, etc. (See figure 3.34). The generator protection relay is normally an integral part of a generator control panel in the switchboard. Similarly, all sections and cables of the power system and consumers are protected in order to avoid over-loading and to reduce the consequences of a failure.
Figure 3.32: Governor for a diesel engine, schematics of speed droop and isochronous control modes.

Figure 3.33: in order to obtain reactive load sharing.
Typically, the protection functions for a feeder to e.g. a motor will contain functions for overload prevention, isolate or alarm at earth fault, and isolate at short-circuit. For short circuit and overload protection, the protection should be adjusted for the equipment’s maximum fault ratings, as well as adapted to protection relays further up or down in the system in order to secure the selectivity that is required by class society.

**Propulsion controller**

As earlier explained, the propulsion is normally speed controlled. The propulsion controller will then keep the reference speed as far possible within the speed and torque limitations, and dynamic capability.

The propulsion controller will normally be interfaced with the thruster / propulsion system, the power generation and distribution system and/or power management system, and bridge control systems including remote control joystick, autosail systems, dynamic positioning, etc.

Since the propulsion power normally constitutes the major part of the total load for the power plant, it is also essential that the load reduction functions and blackout prevention functions are highly coordinated with the power plant design and power management functionality. Based on an overall blackout prevention philosophy, and prioritizing of the different consumers, a complete coordination program as shown in Fig. 3.31 can be made. Since response time is critical for blackout prevention, timing of load reduction is essential. Typically, the propulsion controller will contain three levels of load reduction in the blackout prevention philosophy:

- Maximum load limitation from available power calculation is normally received from the power management system. This gives a max kW loading for the motor drive, depending on the power allocation and priority used for the design of the power management system.

- A fast acting, event-triggered load reduction, typically a digital signal forcing the motor drive power to be reduced to a preset reduction ratio, or absolute value. The signal may come from the power management system.
Figure 3.35: Torque, or power control of thrusters and main propulsion not only improve the dynamic performance of the positioning or sailing control functions, but also stabilizes the power taken from the network, and reduces the power disturbances compared to pure speed control and even more compared to pitch control.

- A fast acting, frequency triggered load reduction, only depending on a local frequency measurement at the drive’s line supply. Normally regarded as a last protection against under frequency trip due to generator overload in the blackout prevention philosophy.

Safety and monitoring functions of propulsion related equipment and auxiliaries are to some extent required by classification society. Depending on the overall system design, monitoring and shutdown functions may be included as part of the propulsion control system, or in the integrated automation system, or in a combination.

There has been shown that utilizing torque control for propulsion and thrusters (Lauvdal et al. [162]) will have a stabilizing impact on performance and network disturbances in sailing and station keeping conditions. Figure 3.35 shows how the power drawn from the network will stabilize with a torque control approach compared to speed control, by reducing the effects from sea current and wave impact on the propeller’s thrust characteristics.

3.5 Electric Propulsion Drives

3.5.1 Introduction

Variable speed drives has been in industrial use since in many decades, but first at the end of the 1960’s by use of power semiconductors. At the beginning, DC motors where the most feasible alternative for propulsion control, but during the 1980’s, AC motor drives became industrially available, and commercially competitive. Since then, almost all new deliveries of electric propulsion are based on one of the AC drive topologies.

3.5.2 Variable Speed Motor Drives

The most commonly used converters for motor drives are, in the following order:
• Voltage source inverter (VSI) type converters for AC motors, normally asynchronous motors
• Cycloconverters (Cyclo) for AC motors, normally for synchronous motors
• Current source inverter type (CSI) converters for AC motors, normally synchronous motors
• DC converters, or SCR (Silicon Controlled Rectifier) for DC motors

The topic will be approached in opposite order, due to the fact that the DC converter is the simplest and easiest to understand, while the other ones have a more complex configuration, but building on much of the same building blocks as the DC converter.

Full-bridge thyristor rectifiers for DC motor drives (SCR)
The most commonly used DC motor is the shunt motor, which has separately supplied field winding and armature (rotor) winding. The armature current is transferred from the stationary terminals to the rotor by use of brushes connected to the rotating commutator. In practice, the armature current also flows through some additional stationary windings, which aids the commutation of current between the segments of the commutator, but this effect is not regarded here.

In a shunt DC motor the induced armature voltage is proportional to the magnetic field and rotational speed. The magnetic field is a function of the field current, and because of saturation effects, they are in practice not proportional. However, if neglecting the saturation, the armature voltage is:

\[ V_a = k \Phi I_f n \approx k K_\Phi I_f n = K_V I_f n, \]  

where \( K_V \) is the induced voltage constant, \( I_f \) the magnetization (field) current, \( n \) is the rotational speed, \( K_\Phi \) and \( K \) proportional constants and \( \phi \) is the motor flux. The developed torque is proportional to armature current and magnetic field

\[ T = k I_a \Phi I_f \approx k I_a K_\Phi I_f = K_T I_a I_f, \]  

where \( K_T \) is the torque constant and \( I_a \) the armature current.

Since the DC motor must be supplied from a DC source with limited voltage, field, and armature currents, the characteristic boundary of operations will be as shown in Fig. 3.36.

The operation is divided in a constant torque region, characterized with a constant field current, and a field weakening region where the field current is reduced to maintain the maximum armature voltage level as speed increases. Hence, the maximum torque boundary is in principle constant in the constant torque region, and inverse proportional to speed in the field-weakening region.

However, in the lower speed region, the armature current normally must be limited to avoid burning of the commutator, and in the higher speed region, it must be reduced to avoid flashing between the segments of the commutator. These limitations are indicated in the diagram.

In the most common high-power applications, a full-bridge thyristor rectifier (Fig 3.37) feeds the DC motor with a controlled armature (rotor winding) current. Similarly, the field winding is excited with a regulated field current. The torque is controlled accurately and with low ripple if the armature inductance is high, but this, on the other hand, reduces the dynamic performance since the time constant of the armature increases. In this topology, the DC voltage on the motor
armature windings is controlled by phase shifting the thyristors’ conduction interval by the gate firing angle \( \alpha \). The gate firing angle can in principle be controlled from 0 to 180 degrees, and the voltage on the armature windings can hence be regulated from +1.35 to –1.35 times the line voltage. In practice, however, the gate firing angle will not be lower than 15 degrees, in order to ensure controllability of the motor drive also with voltage drops in the network, and limited to 150 degrees to have a so-called commutation margin.

Since the armature current is controlled by use of the firing angle of the thyristor devices, the \( AC \) currents will be phase-shifted with respect to fundamental voltage. The phase angle of the current is almost equal to the gate-firing angle. Since the armature voltage is proportional to the rotating speed, one see that the phase angle, which is approximately equal to the gate firing angle, also will be approximately proportional to voltage and hence rotational speed. In a \( DC \) motor drive, where the speed is varying from 0 to 100%, the power factor will hence vary also from 0 to 0.96 (\( \alpha = 15 \) degrees). A low power factor increases losses in the generation and distribution system and more generators may have to run than the active power of the load apparently would require.

Wear and tear on brushes and commutator is a source of failure and maintenance and also limits the standstill torque performance. When accounting for this and also for the fact that the practical limit for \( DC \) motor drives is \( 2 – 3 MW \), the application of \( DC \) thruster drives is limited, with the exception of retrofits, in which existing installations are reused.
Current source converters

A DC current link fed by a thyristor-controlled rectifier and smoothed by an inductor characterizes the current source inverter (CSI), occasionally referred to as a load-commutated inverter (LCI) or Synchro. This converter is normally used together with a synchronous motor, but can also, with some modifications, be used to drive an asynchronous motor. The asynchronous motor variant was more common in the past, but is rarely seen in new installations.

The synchronous motor is similar to the synchronous generator, with rotating field (magnetizing) windings and three- or six-phase stator windings. Six-phase stator windings must be supplied from a double CSI inverter and is used to reduce the torque harmonics on the shaft.

From the network side, the current source inverter is identical to a full bridge thyristor converter used for DC motor drives, and the characteristics towards the network can very much be considered to be the same. The inverter side, which feeds the motor, has the same topology as the rectifier, and uses the induced voltages from the motor instead of the network voltage.

The thyristor rectifier results in a speed-dependent varying power factor, which is high (0.9) at nominal motor speed, and decreasing toward zero for low speeds. The supply current contains harmonics that must be regarded during the system design and should normally be reduced by use of a 12 pulse, 6-phase configuration.

The DC link current is directed through the motor phases by controlling the thyristors of the inverter stage. A 6-step current waveform is obtained, resulting in motor harmonics and torque ripples. The CSI requires a certain counter-induced voltage (EMF) from the motor to perform commutation. Hence, it is mainly used in synchronous motor drives in which the motor can be run with capacitive power factor.

At lower speeds, typically below 5-10% of rated speed, the EMF is too low to perform a natural commutation. In this speed range, the CSI is run in pulsed mode in which the current is controlled at zero level during commutation of the inverter output stage. Since the current and hence the torque are forced to zero level, the torque pulsation at the motor shaft is large in this operational area. The torque ripple and hence shaft vibrations should be carefully regarded in the propulsion system design to reduce vibrations and acoustic noise. These may have a detrimental effect on geared thrusters operating in DP mode.

The characteristic operating boundaries are shown in Figure 3.39 for the topology in Figure
The CSI is used in large synchronous motor drives; the biggest one supplied is approximately 100 MW.

Cycloconverters

The cycloconverter (Cyclo) is a direct converter without a DC link (see Fig. 3.40). The motor AC voltage is constructed by selecting phase segments of the supply voltage by controlling the anti-parallel thyristor bridge. A 12-pulse configuration with reduced line harmonic is drawn, but the cyclo can also be supplied in a 6-pulse configuration. In 6-pulse configuration, the feeding transformers can be substituted with reactors when the supply voltage matches the inverter voltage.

The motor voltage is controllable up to about one third of the supply frequency (about 20 Hz); thus it is most applicable in direct shaft drives without gear. It has been used for main propulsion systems, including podded propulsion.

The motor voltage contains a lower level of harmonics than the CSI, and the motor power factor may be kept high (unity in synchronous motor drives).

The supply power factor is motor voltage-dependent and is about 0.76 in the field weakening range. The content of line harmonics is speed-dependent and must be carefully regarded in system design when the motor drive is large compared with the installed power.

The operation boundaries are similar to those found in the CSI type of synchronous motor drives, except that the low speed limitations are not present, since the commutation takes place towards the network voltages and not the motor voltages. The cyclo converter has hence been preferred in applications where low speed operation and performance is essential, especially in ice breaking or ice going systems, but also in DP and passenger vessel applications where low speed / maneuvering performance is essential.

The Cyclo is available in a power range of 2 – 30 MW per drive motor.
Figure 3.39: Characteristics operating boundaries for a CSI (LCI) converter fed synchronous motor

Figure 3.40: Cycloconverter drive with input and fundamental output waveforms. The output voltage is constructed by selecting phase segments of the supply voltage.
Voltage source inverters

The VSI (Voltage Source Inverter) converter is by far the most used frequency converter in industrial applications. It gives the most flexible, accurate and high performance drive, and can be used with an asynchronous motor. It can also be used for synchronous and permanent magnet synchronous machines with much better performance than other alternatives. The main limitation of this drive topology has been the availability to high power components, and its competitiveness towards other drive topologies in the high power range. Until recently, the practical limit for these drives was around $8 - 10 \text{MW}$, but as new components become available, this limit tends to be higher, today up to about $25 \text{MW}$.

The VSI is characterized by a rectifier, normally an uncontrolled diode rectifier connected to the network. This rectifies the network voltage, which hence gives a relatively constant DC voltage, which is further smoothed by a capacitor bank in the DC link. The capacitor in the DC link also ensures that high frequency switching ripple from the inverter module does not enter the network. A six-pulse VSI converter fed induction motor drive is drawn in Fig 3.41 and its characteristic operation boundaries drawn in Fig. 3.42.

The rectifier in Fig. 3.41 represents a six-pulse configuration used where the converter is directly connected to the network. The dominant harmonic currents are of the 5th, 7th, 11th, and 13th harmonic order. The harmonic distortion can be further decreased when using 12-pulse configuration with a dual feeding via a three-coil transformer, hence canceling the 5th and 7th harmonics. Where a transformer is necessary for voltage adaptation, the 12-pulse configuration
should normally be used. Using PWM drive and 12-pulse configurations, the resulting harmonic distortion will normally be close to the limits defined by rules and guidelines, but additional means may be required, e.g. filtering.

The topology in Fig. 3.41 is capable of running the motor in both directions. Due to the diode rectified power supply, power can however only be taken from the network, not be fed back to the network during regenerative breaking. The inverter part, feeding the motor, has the capability to also perform regenerative network. If this should happen in the topology shown, the DC link voltage would increase and the components might suffer from damages due to overvoltage. All converters have a built in overvoltage protection, that limits the breaking power if the DC link voltage increases above a safety limit.

In order to be able to regenerate power, e.g. due to a crash-stop maneuver by reversing the propeller speed, as , it is common to build in a transistor controlled resistor bank to the DC link, which is activated before the safety limit for DC link overvoltage. The regenerated power will then be dumped in this resistor. Alternatively, the rectifier can be equipped with a full bridge thyristor rectifier in anti-parallel with the diode rectifier (see next section) or an active front end (similar to the inverter module) can be applied as a rectifier unit in order to feed power into the network.

There are several ways to control the switching elements in order to obtain the desired voltage output to the motor. The most common methodology is to use the PWM (Pulse Width Modulation) in some variant. In its most basic version, a three phase PWM voltage is generated by comparing three sinusoidal reference values to a high frequency triangular signal, as shown in Fig. 3.43. While the sinusoidal reference is higher than the triangular signal, the upper

---

Figure 3.42: Characteristic operation boundaries for a six-pulse VSI converter fed induction motor.
Upper and lower switching elements are switched in opposite orders:
- ON: Upper = on, Lower = off
- OFF: Upper = off, Lower = on

Figure 3.43: Generation of switching pulses to the PWM modulated inverter, and resulting output voltages to the motor phase and between two lines.

switching element in the inverter leg gets a firing signal; the lower is turned off, and opposite when the sinusoidal reference signal is lower than the triangular signal. The voltages from the inverter to the motor terminals follow the same pattern; with instantaneous values equal positive and negative voltage levels of the $DC$ link for positive and negative gate control signal respectively. The line voltage, which is what influences the motor, is then the difference between two phase voltages as shown in Fig. 3.43.

As an alternative to such PWM methods, there are vector modulation techniques, and direct modulation techniques as in the direct torque control (DTC), where the gate firing signals are generated directly by calculating which of the 8 possible voltage vectors (including two zero vectors) to be applied to the stator windings.

There are various methods to implement the motor controller, which attempts to create input to the motor which gives the desired torque:

- Scalar control: Scalar control is the simplest and first applied technology for control of asynchronous motors. It was possible to implement in analog electronics, which was the
only feasible in the earlier days of motor control. The scalar control is based on the stationary model of the asynchronous motor, and from this the corresponding voltage and frequency was calculated which would give the desired torque or speed in the motor. The disadvantage is that the model is only valid in stationary condition, and that the model parameters are highly dependent on temperature, frequency, etc. Hence, the scalar methods have a poor dynamic performance, with a poor utilization of the motor capacity.

- **Rotor flux vector control:** This methodology was developed in the late 60’s by the German scientist Blaschke. The method is based on a model of the motor voltage, fluxes, and currents referred to vectors in a rotating coordinate system. With the coordinates oriented in synchronism with the rotating flux in the rotor winding, the current vector’s components are de-coupled in a flux component and a torque component, similar to the DC motor’s field current and armature current. The method requires computer capacity far above what was available when the control methodology was developed, and this method found its commercial application in the early to mid 80’s. A disadvantage is still that the model required to do the vector transformation, contains parameters highly varying, especially the rotor resistance which depends on temperature. In order to obtain a good dynamic performance, the rotor resistance should be adapted on-line or the temperature should be measured. Figure 3.44 shows a schematic diagram for this control scheme.

- **Advanced stator vector control:** The same de-coupling of flux and torque control can be achieved by using a model of stator fluxes and currents in a stator oriented coordinate system. This model can be made independent on the highly varying rotor parameters, but requires a much higher computer capacity of the controller. By simulation, this methodology was known as early as the late 80’s. Then in the mid 90’s the method, also known as direct torque control (DTC) was made commercially available. The mathematical model of the asynchronous motor must be solved with typically 40kHz sampling frequency for accurate control, and will then not be able to estimate the electrical quantities of the motor, but also the mechanical speed of the motor. This enables the use of tacho-less drives in most applications, which is regarded as a great enhancement of the reliability of the system.

Fig 3.45 shows the limitations of the VSI type asynchronous motor drive. The voltage limitation on stator voltage is given by the maximum output voltage from the inverter by the given, constant DC link voltage. The inverter unit or motor max current gives the limitations in stator current and torque. Normally, this means that the motor is limiting the current and torque in continuous load operations, and the inverter current at intermittent load. There is also another limitation; labeled with “pitching moment limitation” that is a characteristic of the induction motor itself. It normally occurs at 150 to 200% of nominal speed, which normally is outside of the operational speed range of a propulsion motor.

The topology in Figure 3.41 is capable of running the motor in both directions. Due to the diode rectified power supply, power can however only be taken from the network, not be fed back to the network during regenerative breaking. The inverter part, feeding the motor, has the capability to also perform regenerative network. If this should happen in the topology shown, the DC link voltage would increase and the components might suffer from damages due to overvoltage. All converters have a built in overvoltage protection, that limits the breaking power if the DC link voltage increases above a safety limit.
Figure 3.44:

Figure 3.45: Characteristic maximum operating boundaries for a VSI asynchronous motor drive.
Figure 3.46: Breaking capability (4-quadrant operation) is required for propulsion drives, where the crash stop maneuver is accomplished by speed reversal of the propeller.

In order to be able to regenerate power, e.g. due to a crash-stop maneuver by reversing the propeller speed, as seen in Figure 3.46, it is common to build in a transistor controlled resistor bank to the DC link, which is activated before the safety limit for DC link overvoltage. The regenerated power will then be dumped in this resistor. Alternatively, the rectifier can be equipped with a full bridge thyristor rectifier in anti-parallel with the diode rectifier (see next section) or an active front end (similar to the inverter module) can be applied as a rectifier unit in order to feed power into the network.

Under bollard pull condition the power flow will be positive, i.e. from network to motor, in stationary. Dynamically, there might be a dynamic breaking moment to stop or reduce the speed of the propeller, if the speed reduction is faster than the load from the water resistance may

Figure 3.47 illustrates the four quadrants of operation in a torque-speed diagram. The topology in Figure 3.41 may only operate in quadrants I and III. With resistor breaking or active front end, the VSI converter may operate in all four quadrants. This is typically needed for main propulsion, winches, elevators, etc., while thrusters normally use a two quadrant converter.

The six-pulse converter in Fig.3.41 does not draw a sinusoidal current from the network. In order to reduce the distortion, it is common to use a 12 pulse configuration as is shown in Fig 3.48. This is fed from a transformer with 30 degrees phase shifted secondaries (Ddy transformer) and with series or parallel connection of two six-pulse diode rectifiers, the resulting distortion of the primary currents will be significantly reduced. Similarly, one can also use 18-pulse (three diode rectifiers and four winding transformer) or 24-pulse (four diode rectifiers and a five-winding transformer) to further reduce the distortion. A 12-pulse configuration will normally be enough to bring the distortion down to an acceptable level. Harmonic distortion will be described more in detail in a later section.

Medium voltage converters are usually a modified version of the one shown in 3.41. Due to the increased voltages, the DC link voltage must be divided on more components in series in the inverter part, as shown in Fig. 3.49. This shows a three-level converter, since the output voltage now can be varied between three levels, +, 0, and -, while the two-level converter in Fig 3.41 can only be varied between + and -. The three-level converter will also give lower current distortion in the feeding currents to the motor for the same switching frequency as a two-level converter. This means that switching frequency can be kept lower, given lower losses in the component, with an acceptable current, and hence torque ripple, as shown in Figure 3.50.
Figure 3.47: The four quadrants of operations with operational limits for VSI converter.

Figure 3.48: 12 pulse diode rectifiers. Series connection is often used in medium voltage converters, parallel in low voltage converters.
Figure 3.49: a) Three-level inverters are often applied in medium voltage converters, since the voltage stress on each component will be lower and the harmonic distortion of the motor current will be lower at the same switching frequency; see Fig 5.13. b) Shows how current may flow to obtain, from left; + voltage, 0 voltage, and – voltage to the motor.

medium voltage, 12 pulse, three-level VSI drive for induction motor drives with water cooling is shown in Figure 3.51.

Other converters

In addition to the most used topologies mentioned here, other variants are occasionally also seen. Examples are CSI with PWM current output, step-wave with multiple transformer output and extremely low motor voltage distortion, and VSI Pulse Amplitude Modulated (PAM) converters with a thyristor-controlled DC link voltage and a 6-pulse voltage output. The use of these technologies is limited and normally only seen in special applications.

Comparison of electric motor drive alternatives

The characteristics of a DOL-started fixed-speed CPP unit and a variable speed FPP unit are compared in Fig. 3.52. Because of a lower power factor and higher starting transients more diesel-generators should, in general, be connected to the power network with fixed-speed thrusters than with variable-speed thrusters, and hence with variable speed thruster and propulsion drives:

- Average loading will be higher with less running hours.
- Fuel costs, wear and tear, and maintenance will decrease.
- Smaller dimensioning of the power plant may be achieved.

A thruster drive has two prices: one before and one after installation. The equipment cost of the variable-speed drives will often be higher than that of the constant-speed controllable-pitch propeller. Maintenance cost and fuel consumption will on the other hand be reduced since in DP operation the thruster power is usually only partially utilized.
Figure 3.50: A medium voltage, 3300V IGCT VSI drive for induction motor drives (ABB ACC 6000). Cabinets (from left): 1: Diode rectifiers, 2: Terminal cabinet and control modules, 3: IGCT inverter module, 4: DC link capacitors, 5: Water cooling unit with heat exchanger and circulation pumps. The internal cooling water must be de-ionized (non-conducting) since the components are directly mounted without insulation to the water-cooled heat sinks. In low voltage drives, the power modules are normally isolated from the cooling heat sink, hence normal fresh water can be applied.
<table>
<thead>
<tr>
<th></th>
<th>DOL asynchronous motor + CPP</th>
<th>SCR DC motor drive</th>
<th>Cyclo-(^1) converter</th>
<th>CSI (LCI) (^2)</th>
<th>VSI PWM (^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start-up amps</td>
<td>Typ. 5 x rated current (= 0) (transformer inrush)</td>
<td>(= 0) (transformer inrush)</td>
<td>(= 0) (transformer inrush)</td>
<td>(= 0) (transformer inrush)</td>
<td>(= 0) (transformer inrush)</td>
</tr>
<tr>
<td>Start-up torque transients</td>
<td>Typ. 2-3 x rated torque (= 0)</td>
<td>(= 0)</td>
<td>Up to 50% of rated torque</td>
<td>(= 0)</td>
<td>(= 0)</td>
</tr>
<tr>
<td>Power consumption, low thrust</td>
<td>(\approx 15%) of nominal power</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(= 0)</td>
<td>(= 0)</td>
</tr>
<tr>
<td>Amps at low thrust</td>
<td>45-55% of nominal</td>
<td>(F) (torque)</td>
<td>(F) (torque)</td>
<td>(F) (torque)</td>
<td>(= 0)</td>
</tr>
<tr>
<td>Power Factor - full load</td>
<td>(&gt; 0.85)</td>
<td>(&gt; 0.9)</td>
<td>(&gt; 0.76)</td>
<td>(&gt; 0.9)</td>
<td>(&gt; 0.95)</td>
</tr>
<tr>
<td>Power factor variation with load (cos&lt;sub&gt;\theta&lt;/sub&gt;)</td>
<td>0.15 ... 0.85 (non-linear)</td>
<td>0 ... 0.9 (prop. speed)</td>
<td>0 ... 0.76 (prop. speed)</td>
<td>0 ... 0.9 (prop. speed)</td>
<td>(&gt; 0.95) (c constant)</td>
</tr>
<tr>
<td>Dynamic response (power, torque)</td>
<td>3-5 sec (pitch control)</td>
<td>(&lt; 100) ms</td>
<td>(&lt; 100) ms</td>
<td>Slower</td>
<td>(&lt; 50) ms</td>
</tr>
<tr>
<td>Torque ripple</td>
<td>None</td>
<td>Smooth</td>
<td>Smooth</td>
<td>Pulsating</td>
<td>Smooth</td>
</tr>
<tr>
<td>Zero-thrust crossing</td>
<td>Smooth if negative thrust allowed</td>
<td>Discontinuously</td>
<td>Smooth</td>
<td>Pulsating</td>
<td>Smooth</td>
</tr>
<tr>
<td>Efficiency at full load</td>
<td>High</td>
<td>Lower</td>
<td>High</td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td>Harmonic distortion:</td>
<td>None</td>
<td>(F) (torque)</td>
<td>(F) (torque)</td>
<td>(F) (torque)</td>
<td>(= 0) (power)</td>
</tr>
<tr>
<td>Short circuit contribution</td>
<td>Typ. 5 x nominal power</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Motor matching required</td>
<td>-</td>
<td>Some</td>
<td>Some</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Commutator</td>
<td>No</td>
<td>Yes</td>
<td>No (sliprings)</td>
<td>No (sliprings)</td>
<td>No</td>
</tr>
</tbody>
</table>

\(^1\) With brushed synchronous motor  
\(^2\) With brushed synchronous motor  
\(^3\) With cage induction motor

Figure 3.52: Comparisons of drive alternatives. Low thrust equals low speed in variable speed drives and small pitch in fixed speed drive for CPP propeller.
3.6 System Design

3.6.1 Introduction

One of the more experienced class surveyor has stated: “Nobody should make the mistake of assuming that diesel electric propulsion is easy”. On the other side, diesel-electric propulsion has proven to be reliable and robust when all aspects of design and engineering are sufficiently carefully examined.

The main difference between the marine power system and a land-based system is the fact that the marine power system is an isolated system with short distances from the generated power to the consumers. The amount of installed power gives special challenges for the engineering of such systems.

The design and engineering phase can simplified be illustrated by Fig. 3.53. Even before the vessel concept design starts, a market assessment based purpose and requirement specification for the vessel should be the made as the basis for the design work.

The system design and engineering can be divided in two phases, however, the first phase is not fully independent on the possibilities and constraints of the second;

1. Design, conceptual and detailed: In these phases, the vessel capability must be specified and analyzed in order to define the amount of thrust required for sailing, maneuvering, and station keeping, as applicable for the intended operations. Based on the goals and objectives for the vessel operations, the type of propulsion and propulsion / thruster units, their rating and location on the vessel should be determined, as well as the most optimal configuration and splitting of the power generation and distribution system. The design phase will result in a set of technical specifications for the vessel, which is the basis for the further engineering work.

2. Engineering, system and detailed: During these engineering phases, several analytical and numerical calculations have to be performed in order to achieve safe and reliable operation, in common described as standard network analysis or electrical power system studies:

   • Load flow calculation.
   • Short circuit calculations.
- Ground fault calculations.
- Relay coordination study.
- Harmonic analysis.
- Voltage drop calculation of inrush of transformers and starting of motors.

Dependent on system configuration and vessel application the following extended analysis can also be required, or necessary:

- Transient analysis of network behavior after disturbance, e.g. short circuit.
- Reliability or failure mode analysis.

A thorough and precise work in this phase is essential for safe, reliable, and cost efficient operations, and flexibility for future upgrades and modifications of the system later during the life time of the vessel. Such work has been made much more simple and accurate by use of computer aided engineering and design tools, and there is a whole range of such available and in daily use, such as (e.g. EDSA, POWER TOOLS, SIMPOW, SIMSEN, etc.).

3.6.2 Life Cycle Cost Assessment of Conceptual Design

The starting point for the conceptual design of the electric power generation and propulsion system is based on the intended operation and operating profile of the vessel which is the results of the vessel and hull design work.

If the vessel has a relatively flat operating profile, meaning that propulsion power is close to constant for most of the life time, e.g. a VLCC tanker in fixed charters, electric propulsion will normally not be economically feasible unless there are other requirements which makes it beneficial. Such can be; large power requirements for other processes, high maneuverability, redundancy, low noise and vibration, etc.

For vessels with more variable operation profile, as shown in Figures 3.54 and 3.55, diesel electric propulsion may be profitable, by pure fuel and maintenance savings, or in combination with increased income.

An accurate comparison of different concepts should be accomplished by use of a Life Cycle Cost assessment (LCC). There are several ways to do this, and NORSOK has suggested a method, intended for use in offshore industry, but it can also be applied for ships. The main elements of the LCC are divided in CAPEX (Capital Expenditures) and OPEX (Operation Expenditures), and are listed in Figure 3.56.

Although a life cycle cost of the complete vessel and its operations ought to be done for a fair and precise comparison of different concepts, this is too seldom done, mainly because of lack of competence, lack of reliable data or lack of time and resources at this stage, or a combination. It is likely to believe that conventional propulsion system is more often selected as a consequence of this, than opposite.

3.6.3 Standard Network Analysis and Electrical Power System Studies

Load flow calculations

The aim of the load flow and short circuit calculations is to determine whether the thermal and mechanical stresses on equipment, such as generators, cables, switchboards, and transformers, is below the maximum design values, under normal as well as contingency conditions.
Figure 3.54: (Left) Operating profile for a cruise vessel (hours per year) and (Right) for a shuttle tanker in the spot market (per day over a year).

Figure 3.55: Operating profile for a field support vessel.
Life Cycle Cost, LCC = CapEx + OpEx

### Capital Expenditures

<table>
<thead>
<tr>
<th>Description</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design and administration cost</td>
<td>The total engineering and project administration cost from the project start to operation.</td>
</tr>
<tr>
<td>Equipment and material purchase cost</td>
<td>The total purchase cost associated with the system.</td>
</tr>
<tr>
<td>Fabrication cost</td>
<td>The total fabrication cost associated with the system.</td>
</tr>
<tr>
<td>Installation cost</td>
<td>The total cost of installing the systems and equipment.</td>
</tr>
<tr>
<td>Commissioning cost</td>
<td>The total cost to commission, and when necessary, certify, the installed systems and equipment.</td>
</tr>
<tr>
<td>Insurance spares cost</td>
<td>The total purchase cost for the initial spares holding for the systems and equipment, necessary to obtain the required system regularly.</td>
</tr>
<tr>
<td>Reinvestment cost</td>
<td>The total cost to remove, refurbish or purchase, install and commission systems and equipment that is predicted to exceed its design life during the life of the facility.</td>
</tr>
<tr>
<td>Finance costs</td>
<td>Finance costs during construction</td>
</tr>
</tbody>
</table>

### Operational Expenditures

<table>
<thead>
<tr>
<th>Description</th>
<th>Description</th>
</tr>
</thead>
</table>
| Man-hour cost                      | Man-hour cost is defined as the cost of the needed man-hours per year to operate and maintain the facility/equipment:  
  - Fixed crew.  
  - Workload dependent crew.  
  - Contractor.  
  - Vendor. |
| Spare parts consumption cost       | The total cost of spare parts and consumables over the design life of the facility and systems, necessary to complete the predicted work load for all maintenance actions (i.e., preventive maintenance, corrective maintenance and servicing). |
| Logistic support cost              | The total logistic support cost necessary to support operation and maintenance requirements for the facility and system (e.g., supply boat, diving support vessel, helicopters). |
| Energy consumption cost            | The total energy consumption cost for the facility and systems. It shall include the cost of fuel required to generate the power and associated CO2 tax. |
| Insurance cost                     | The total cost related to insurance for the facility and systems. |
| Onshore support cost               | The total cost of the required onshore support services and administration. |
| Cost of deferred production        | The total cost of deferred production due to probability of failure of system and equipment. |

Figure 3.56: Elements in NORSOK’s Life Cycle Cost assessment.
It also gives information about the setting of transformer tappings and voltage regulators that ensures that the voltage levels on the different distribution buses and load terminals are within permitted stationary deviation limits.

Load flow calculations are performed to find stationary values of loads in the generation and distribution network, and since the network in ships normally are radial, it is a fairly straightforward exercise. It is normally done prior to the short circuit calculations in order to obtain its starting values.

**Short circuit calculations**

Short circuit calculations are done to ensure that fault current in short circuits does not exceed breaker’s and other equipment’s maximum ratings. Under short circuit, the mechanical stresses on bus-bars and cabling becomes much higher than under normal operation, and class rules and standards defines certain limits that the equipment shall be designed for.

A typical short circuit current waveform for a generator is shown in Figure 3.57. As seen, the initial current has a significant DC component, which together with the sub-transient short circuit current may give a high peak value typically in the order of 10 times nominal current for the generators. The short circuit currents are being reduced, as the DC component decays rapidly, typically with a time constant of $20 - 100\,\text{ms}$. After $300 - 500\,\text{ms}$, the transient short circuit current is typically reduced to $3 - 5$ times nominal generator current, and dependent on the system design, this is normally the breaking current for a circuit breaker for a branch.

A sustained short circuit current will after a second or more, reach a stationary value, which according to rules and regulations shall be at least three times nominal current for a generator in order to detect faults reliably, for clearing faulty branches. The short circuit current is found by numerical simulation or by analytical methods as defined in IEC.

**Ground fault calculations**

If one phase is short-circuited to ground, there will flow a ground fault current. Its magnitude depends heavily on the method of system grounding, which are:

- Ungrounded: Ground fault currents will still flow due to capacitive coupling between
healthy phases and ground, typically in the order of a few amps in a ship installation. Required for tankers by SOLAS.

- Low resistance grounding: Ground fault currents will flow through the ground fault, with a low resistance ground resistor limiting the fault current to not less than 100A.

- High resistance grounding: Ground fault currents will flow through the ground fault, with a high resistance ground resistor, limiting the fault current typically less than 20A.

- Bolted ground: Create a high ground fault current, in the order of a short circuit current.

- Coil grounding: With a proper tuned coil grounding, the fault current is theoretically very low. Not commonly used in ships, mainly since the network configuration varies, and tuning of the coil is impractical.

System grounding in ships are normally either low or high resistance grounding of system neutral point or, ungrounded. It might also be a combination of these in different parts of the distribution system.

Ground fault calculations are done to ensure that the fault current is low enough to reduce the risk of damaging equipment, and to determine the levels needed to adjust ground fault detection relays.

For some ships (especially tankers), there might be desired or required to operate with isolated neutral point in order to reduce fault currents to a minimum. Due to system capacitance, a fault current with a typical magnitude of some Ampere will still flow through the ground fault. The power system may be allowed to continue operation with such low fault currents until it is possible to disconnect and repair faulty parts without large disturbances in operations. The disadvantages with this grounding method is that one can experience high overvoltages due to resonance in the fault current circuits, and it might also be difficult to identify in which branch the fault current occurred.

A high impedance neutral point grounding limits the ground fault currents typically to less than 20A. Also with this fault current it might be allowed to continue operation with a fault for a limited period of time. The grounding resistor will reduce the risk of resonance oscillations and it is easier to detect and disconnect the faulty branch of the power system. High impedance neutral point grounding is normally the preferred method in medium voltage systems.

Low impedance neutral point grounding gives high ground fault currents and a ground fault must be cleared by disconnecting faulty parts immediately (typically < 200ms). Compared to isolated and high impedance grounded neutral point, this method will reduce the voltage stress on the healthy phases during the fault.

**Relay coordination/selectivity study**

Each feeder in and out from a switchboard is equipped with protection relays or fuses for detection and disconnection at short circuit, sustained overload conditions, and ground fault. Ground faults with low fault currents may be accepted for continuous operation.

An over-current / short circuit relay is typically adjustable by level of fault currents and time to disconnect. Fuses have correspondingly a certain current-time characteristic selected for the application. A normal load condition shall not initiate a disconnection. If the load current is higher than the defined normal condition, i.e. in overload, the relay starts a time counter, and
disconnects after a certain, and preset time delay. If load current is even higher, the relay starts
another time counter, that disconnects the branch after a shorter time period, defined by the
selectivity study in a way that protection devices in lower parts of the system shall be activated
first, and protection in higher parts later.

Adjusting these settings or selecting of fuses shall according to rules and regulations clear
any fault selectively, by disconnecting a minimum of the distribution system limited to the parts
that are directly affected by the fault. Deviations might be necessary, if the consequences are
regarded not critical.

**Harmonic distortion**

The harmonic distortion level may be significant in electric propulsion systems, as the main loads
usually are variable speed propulsion/thruster drives with frequency converters. Harmonics and
harmonic analysis are described in a separate section.

**Voltage drop calculations**

At start-up of heavy motor consumers or energizing large transformers, the start-up transient
current may be several times larger than the nominal rated current. For a motor, typically 5-8
times higher, and for a transformer, up to 10-12 times higher.

A typical motor start transient with a direct on-line (DOL) started motor is shown in Figure
3.58. The motor starting current, is similarly to a short circuit, a complex time varying curve
containing a $\Delta$ term, and transient as well as stationary terms during the acceleration time.
The time for acceleration to full speed is determined by the motor rating and the load curve of
the motor.

The associated voltage transient is shown as a nearly instantaneous maximum drop imme-
diately after the motor is connected. Then, the AVR starts to increase magnetization to com-
penstate for the increased load currents of the generators. When the motor reaches the pitching
moment, the stator currents quickly reduce and since the generators now are over-magnetized for
the new load, a certain voltage overshoot occur. High start-up currents may cause a significant
voltage disturbance in the network, and class requirements set a limit for acceptable transient
voltage variations. This limit is typically $-15\%$ and $+20\%$ (DnV).

There are analytical methods for calculating such voltage drops, though the most accurate
result is found by numerical simulations.

In order to not exceed the required voltage variations, there might be a need to adjust the
characteristics of the generator or the large consumers, or introduce means to reduce the start-
up transients. For motors, such means could be soft-starting devices, star-delta starters, or
auto-transformer start. For transformers, pre-magnetizing could be evaluated. Generators with
low transient reactance will also give a lower transient voltage drop.

### 3.6.4 Extended Analysis and Studies

**Transient analysis**

In addition to the fault calculations and voltage drop calculations mentioned, there are occa-
sionally a need for more thorough analysis of the transient behavior of the network during and
after clearing a fault.
Figure 3.58: DOL (direct on-line) motor start current and voltage transient.

Typically, such analysis includes voltage and frequency stability (i.e. will voltage and frequency of the generators be re-established after the fault is cleared?) and re-acceleration of motor loads (will essential motors be able to accelerate without tripping after the fault is cleared?). Such analysis are extensive and require accurate modeling of the network and regulators for voltage and frequency in order to be reliable. It is normally only done when regarded necessary, and in systems with large motors.

**Reliability analysis and FMEA**

Especially for ships where availability of the power system is essential and with redundancy requirements, there is normally a request for reliability analysis or Failure Mode and Effects Analysis (FMEA). These shall identify the consequences of faults in components or system, their criticality and assess the probability for such events. Its objective is to identify critical characteristics that can be improved in the design or specially considered for operations.

The reliability analysis is a quantitative approach to find the likelihood for certain fault scenarios, such as loss of parts of propulsion power, blackout, loss of positioning capability, etc. It is normally accomplished by use of a fault three analysis and calculation, known from statistics theories.

The FMEA is a more qualitative approach, focusing on identifying the consequences of certain fault scenarios, with a qualified description of how such scenarios are detected, and avoided / compensated for.

Typical elements for each scenario are description of:

1. Initial conditions.
2. Failure mode of components.
3. Effects on system.
4. Consequences (No, low, high, severe, catastrophic).
5. Occurrence (Rarely, seldom, normal, frequent).
6. Detection and corrective actions.
7. Criticality (acceptable, not acceptable).

The result is normally input to an overall FMEA analysis for the plant, documenting the need for design improvements, operational instructions, etc.

### 3.7 Harmonic Distortion

When the connected load in a network is not linear, i.e. not draws sinusoidal currents, the load currents will distort the sinusoidal voltages. This deviation from a sinusoidal voltage or current wave form is called harmonic distortion.

Distortion of the voltage wave form may lead to:

- Accelerated aging of insulation material: Increased power dissipation (losses) in equipment connected to the network, such as generators, motors, transformers, cables, etc., from the harmonic currents, may cause overheating and deterioration of the insulation, and reduced life time of the equipment.

- Overloading of electronic equipment: Increased load current of electronic equipment that has been designed for sinusoidal voltage supply, may cause overheating and malfunction of this equipment.

- Malfunction: The distorted wave form may cause electromagnetic interference or erroneous measurement signals if the equipment is not designed for the actual distortion. It is particularly necessary that measurement systems of monitoring and protection devices are made for true rms measurements in order to function properly.

The harmonic distortion level may be significant in electric propulsion systems, as the main loads usually are variable speed propulsion/thruster drives with frequency converters.

Rules and regulations normally give guidelines or requirements that limit the harmonic distortion in a ship network. However, these limitations are not a guarantee for proper functionality. It is therefore necessary to be able to predict harmonic distortion, evaluate the effects, and perform the proper means to manage the voltage distortion, without functional faults over the life time of the installation.

#### 3.7.1 Harmonics of VSI Converters

Any periodic wave form can, in general, be expressed by a Fourier series, as the infinite series of sinusoidal components and a DC term

\[
 u(t) = u_{dc} + u_1 \sin \omega_1 t + u_2 \sin (2\omega_1 t + \varphi_2) + u_3 \sin (3\omega_1 t + \varphi_3) + \ldots \\
+ u_h \sin (h\omega_1 t + \varphi_h) + \ldots 
\]  

(3.10)
Some of the terms can be zero, such as the DC terms in most AC applications and the triple harmonics in symmetric three-phase systems which are isolated from ground.

The frequency converters are inherently non-linear and the currents to a motor drive are not sinusoidal but distorted by harmonic components of generally any order, but as we will see, most of the frequency components are zero under ideal conditions.

When analyzing the harmonic distortion of the network supply, one can normally, at least initially, disregard the motor side behavior, by assuming ideal de-coupling between the network and motor sides by the DC link. In Figure 3.41, a VSI converter with diode rectifier and smoothing DC inductor and capacitor has been shown in a 6-pulse configuration, and a 12-pulse configuration in Figure 3.48. If the smoothing DC components are large, the current wave forms will approach the ideal shapes as shown in Figure 3.59.

By observation, one can see that the current into the 12 pulse rectifier is equal to the 6-pulse rectifier, but the phase shift of the Y-connected transformer secondary will shift all voltages and currents by $30^\circ$, compared to the $\Delta$-connected secondary. The current waveforms shown, are those of the transformer windings.

Assuming that converter and transformer are symmetrically designed and output stage of converter is assumed to be de-coupled from rectifier current, only the characteristic harmonic components is present in the input currents to the line supply of the frequency converter. For a 6-pulse converter these are

$$h = 6nx \pm 1, \quad n = 1,2,\ldots$$

$$\downarrow$$

$$h = 5, 7, 11, 13, \ldots$$

In a 12-pulse converter, multiples of sixth ($\pm 1$) harmonics, which are present in the secondary and tertiary windings of the feeding transformer, will due to the $30^\circ$ shift be cancelled in the primary windings and thus the remaining harmonic current components will be of order

$$h = 12nx \pm 1, \quad n = 1,2,\ldots$$

$$\downarrow$$

$$h = 11, 13, 23, 25, \ldots$$

Figure 3.59: Ideal waveforms in a 6- and 12-pulse VSI converter.
The Total Harmonic Distortion (THD) is a measure of the total content of harmonic components in a measured current, \( THD(i) \), or voltage, \( THD(u) \)

\[
THD(i) = 100\% \times \frac{\sqrt{\sum_{h=2}^{\infty} i_h^2}}{i_1}, \tag{3.15}
\]

and

\[
THD(u) = 100\% \times \frac{\sqrt{\sum_{h=2}^{\infty} u_h^2}}{u_1}, \tag{3.16}
\]

where \( u_1, i_1 \) are the fundamental \textit{rms} value of the voltage and current, and \( u_h, i_h \) are the \textit{rms} value of the \( h^{th} \) harmonic of the voltage (or current).

### 3.7.2 Harmonics of CSI Converters

For a CSI converter, the characteristic harmonics will be similar to a VSI converter. However, the de-coupling between the line supply and the motor sides are not as ideal as for the VSI, and the harmonics of the line side currents are strongly influenced by the motor side harmonics. In addition to the pure harmonics, a CSI drive also generates non-integer harmonics to the power network. Non-integer harmonics are interfering components at frequencies that are not exact multiples of the system frequency.

In an CSI drive these non-integer harmonics are due to the DC pulsation frequencies caused by the machine converter and are therefore synchronous with the motor frequency according to the following formula

\[
f_i = hf_N \pm pf_M, \tag{3.17}
\]

where

- \( f_i \) Non-integer harmonic component.
- \( h \) Characteristic harmonic component from drives (1, 5, 7, 11, 13 etc.).
- \( f_N \) Network frequency.
- \( p \) Pulse-number of the drive.
- \( f_M \) Machine frequency.

The amplitude of the non-integer harmonic components are mainly determined by the size of the DC inductor, i.e. the larger inductor the lower amplitudes. Secondly, the amplitudes are in general much smaller than the integer harmonic components.

### 3.7.3 Harmonics of Cycloconverters

For Cycloconverters the harmonic component content of the input current will be a function of both input frequency and output frequency, for a 6-pulse cycloconverter

\[
f = (6 \times n \pm 1) f_i \pm (6 \times p) f_o, \quad n = 1, 2, \ldots, \quad p = 0, 1, \ldots \tag{3.18}
\]

where \( f_i \) is input fundamental frequency and \( f_o \) is output fundamental frequency. As seen, also here there is a rich content of both harmonics and no-integer harmonics in the current, and thus voltage waveforms.

The amplitude of the non-integer harmonics are normally significantly high, and it is normally regarded difficult to establish an efficient tuning of a passive filter to reduce the harmonic level with Cycloconverter loads.
3.7.4 Limitations by Classification Societies

The classification societies have quite recently started to define limitation on allowable THD for voltage waveforms on the switchboards. For example DnV says that for distribution systems the THDv shall normally not exceed 5 %, unless being documented that affected equipment is designed and tested to the actual conditions. It is not defined how documentation and testing shall be done.

A harmonic analysis study is usually required for documenting the harmonic distortion level and to find dimensioning criteria for generators, transformers, and if necessary, filters for reduction of harmonic distortion.

3.7.5 Harmonics of Ideal 6- and 12-pulse Current Waveforms

For the idealized current waveforms in Figure 3.59 (a), one can establish the harmonic spectrum by the following relation (since the wave-form is an odd function with average zero)

\[ i_h = \frac{2}{T} \int_{-T/2}^{T/2} i(t) \sin \frac{2h\pi}{T} t \, dt. \quad (3.19) \]

Using this relation, one find the following spectrum, where \( \hat{I} \) is the amplitude of the current

\[
\begin{align*}
    i_1 &= \hat{I}, \\
    i_2 &= i_3 = i_4 = 0, \\
    i_5 &= i_1, \\
    i_6 &= 0, \\
    i_7 &= i_1, \\
    i_8 &= i_9 = i_{10} = 0, \\
    i_{11} &= i_1, \\
    i_{12} &= 0, \\
    \vdots
\end{align*}
\]

That is

\[ i(t) = \hat{I} \sum_{h} \frac{1}{h}, \quad h = n6 \pm 1, \quad n = 1, 2, 3, 4, \ldots \quad (3.20) \]

Figure 3.60 shows the result of this series, where the terms up to 37\textsuperscript{th} harmonics are included, showing how the resulting waveform converges towards the original six-pulse shape.

In the 12 pulse current waveform, the harmonics of order 5, 7, 17, 19, etc., cancel due to the 30 degree phase shift of the three-winding transformer. These harmonics will flow in the transformer windings, but with opposite phase, in the secondary windings of the transformer, and by the summing, they will circulate inside the transformer only, and not flow into the network. The total harmonic distortion of these current wave forms can be found by the relation given in (3.15), which yields THD\((i)\) about 30% for the 6-pulse current, and 15% for the 12-pulse current.
Figure 3.60: Harmonics up to 37th of a six-pulse current waveform.

Figure 3.61: Characteristic harmonics of a six and twelve pulse current wave-form.
These are ideal current waveforms. In practice, impedance due to inductance, resistance, and capacitance alters the current shape, as seen in Figure 3.63. The corresponding harmonic spectrum that can be measured in a typical installation with VSI converters are shown in Figure 3.62.

It is obvious that the characteristic harmonics are lower than could be expected from ideal curves, and that non-characteristic harmonics, here $5^{th}$, $7^{th}$, etc., occur due to non-ideal transformers and uneven load distribution of the parallel connected rectifiers in the 12 pulse converter.

### 3.7.6 Calculating Harmonic Distortion

#### Basics

The harmonic currents drawn by a non-linear load from the network will be distributed in the network and flow through the other equipment in the power network. If regarded to be a current source of harmonic current components, it is obvious that the harmonic currents will flow through the paths with lowest impedance for the harmonics. These are normally the running generators, large motors, or large distribution transformers to other (higher or lower) voltage levels.

There are two types of simulation tools available: time domain simulation and the more commonly applied, which calculates in frequency domain. The benefit of the frequency domain calculation tools is that the time and work for modeling and calculation of large systems is much shorter than for a time domain simulation. However, the accuracy will normally be lower, since one has to decide the harmonic content of the load current, which in reality is dependent on the network configuration and can only be determined by time domain simulation or by equivalent
Figure 3.63: (a): Example of switchboard voltage waveforms for a system with converters with diode bridge rectifiers. THD is approximately 7%. (b): Example of switchboard voltage waveforms for a system with converters with both diode and thyristor bridge rectifiers. THD is approximately 9%.

figures from similar systems. Special considerations should be made for PWM type of controllers and use of passive filters, where time domain simulations are strongly recommended in order to obtain results that are necessary for correct design and dimensioning.

The simulation circuits can normally be assumed ideal, with symmetric supply and neglected impedance in switchboards and cables. In practice transformers and converters are not ideally symmetrical, nor is the network impedance. Further, there must be expected that non-characteristic current components are present. These effects will usually have a negligible effect unless resonance frequencies become excited.

Cable and load impedance’s, especially capacitive components may increase the distortion on low voltage distributions. In network with high voltage distortion one should avoid the use of tube lighting with capacitive compensators.

An example of waveforms of switchboard voltage and currents to the thruster is calculated by the time domain analysis program KREAN and shown in the two figures below. Fig. 3.63a shows a simulation with only thruster drives with diode bridge converters. Figure 3.63b shows a simulation where thruster drives with diode bridge and drilling drives with thyristor bridge are running simultaneous.

**Frequency domain - harmonic injection**

In this method, the nonlinear load is represented by a harmonic current source, injecting harmonic currents to the network. The network in terms are modeled as a system where its various parts, generator, cable, transformer, motors, etc., are modeled with an appropriate impedance model, representing the impedance for the harmonic frequency currents injected by the harmonic current source.

An example of such models is shown in Figure 3.64, with a harmonic current source representing the frequency converter, and impedance models for generator, cable, transformers, and loads, e.g. motors.

By calculating the resulting voltages from the harmonic currents, the harmonic voltages are
Figure 3.64: Impedance model of a network used in frequency domain calculation of harmonic distortion.

found in the branches or points of interest. Summing up these, the harmonic voltage distortion is finally found.

There are several calculation software programs assisting in building up and calculation of harmonic distortion in the frequency domain. Building up large networks are quite simple by library models, and calculation times are short.

The main challenge is to find a good harmonic representation of the converter, especially when using converter types where the harmonic spectrum is highly dependent on the network, such as with VSI converters. Library models are not always to be trusted.

**Time domain - network simulation**

By building up a circuit model of the network, with discrete impedance models, one can perform a time domain simulation of the system. Initial values of voltages and currents are chosen, and after some simulation time, the system has stabilized sufficiently to represent stationary conditions.

By taking one fundamental period of the voltage or current waveform of interest, one can then perform a Fourier transformation and find the harmonic spectrum at any point or branch of the system.

A simplified circuit model for the same system as in Figure 3.64 is shown in Figure 3.65. It is quite obvious that a complex network is cumbersome to model and time consuming to simulate. Time step in the simulation must also be relatively short in order to give accurate results.

The great benefit is that this model gives an accurate calculation of the voltages and currents, and also the harmonic spectrum of the nonlinear loads.
Comparison of the frequency and time domain simulation

Frequency domain calculations are widely used because of the simple modeling and short calculation times. If the harmonic representation of the converter currents is accurate, the results are also accurate. It is not always straightforward to find the harmonic representation, which may strongly be influenced by the network characteristics. Then, a time domain simulation can be used, either for a complete calculation, or for a part of the system that is representative enough to give a good harmonic model of the converter, and feed this results into a frequency domain calculation for the complete system.

To illustrate the potential faults, if comparing the results of the time domain simulation in Figure 3.63 (a) with a corresponding frequency domain calculation of the same system with the ideal harmonic currents of a 12-pulse rectifier in Figure 3.61, the real voltage distortion gives $\text{THD} = 8\%$, while the frequency domain calculation with ideal current waveforms results in 20\%. This difference is of course so large that the result from the latter calculation is use-less for any engineering aspect.

3.7.7 Managing Harmonics

In a vessel with diesel electric propulsion, the frequency converters may constitute up to 80-90\% of the actual load of the generators. Harmonic effects must be considered, and managed, in order to avoid deterioration and malfunction of equipment, and to meet the rules and regulations’ requirement for harmonic distortion levels.

There are certain engineering aspects that may be used to obtain these objectives, discussed in the following.
Generator impedance

The harmonic currents injected to the power distribution system will mainly follow the lowest impedance routes, which normally are the generators.

For the frequencies of interest in the harmonic analysis, the generator sub-transient impedance is used. The $d$- and $q$-axis sub-transient inductance are normally different, especially in a generator with salient poles, and the average of these, $x_{d''}$ and $x_{q''}$ are normally used, alternatively the negative sequence impedance, $x_-$. A generator with low sub-transient inductance is normally larger than one with larger sub-transients. Normally, the value is about 20%, while it is relatively achievable to reduce this to 15% or even lower.

Another effect of lowering the sub-transient inductance is that the short circuit current level increases. One must therefore always make a trade-off with what is desired from harmonic distortion point of view, the equipment rating for short circuit currents, and the associated overall costs.

Converter topology

The different converter topologies give different harmonic distortion. Normally the power rating and application determine the selection, but when possible, a converter with lower harmonic distortion should be selected to manage the overall distortion levels. Normally, a VSI type converter gives the lowest distortion in electric propulsion applications.

Increasing the rectifier’s pulse numbers also give a lowering of the harmonic distortion, but must be trade-off by the associated costs of transformers and converter.

There are also converters with an active front end, constituted by switching elements instead of diodes. This kind of converters give a much more sinusoidal current shape towards the network, similar as to the motor. However, the costs of these products are much higher than with diode rectifiers.

Design of supply transformer

When a transformer feeds the converter, the transformer’s short circuit impedance should be selected high to smoothen the load current and reduce the harmonic content. It must not be selected so high that the voltage drop over the transformer at full load reduces the power capability of the converter below the specified rating. Normally, the short circuit impedance will be selected between 5 and 8%. A typical distribution transformer will rarely exceed 4%.

Also, the transformer should be equipped with a conductive sheet between the primary and secondary windings, grounded with an efficient high-frequency ground strap. This will not influence the lower harmonic transfer from secondary to primary windings, since these are magnetically coupled. For the very high frequencies, typically above MHz range, the coupling is more capacitive, and the grounded sheet will act as a screen for capacitive currents, leading them to ground instead of to the primary. Such screens are normally required to fulfill EMC regulations (Electro-Magnetic Compatibility), and will also aid as a protection against flashover from a high voltage primary to the low voltage secondary if the insulation should fail.
Figure 3.66: Passive filter in a network with a generator. \( Z(\omega) \) represents the resulting frequency between two lines, as experienced by a frequency converter connected to the network. (a) circuit diagram. (b) frequency response.

**Passive filters**

A passive filter consists of inductances and capacitances, and sometimes also a resistance. Figure 3.66 shows the circuit diagram for a first order LC filter. The impedance in one of these branches for a certain frequency \( f \) is (where \( \omega = 2\pi f \))

\[
Z_{\text{filter}}(\omega) = j\omega L + \frac{1}{j\omega C} = j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)
\]

(3.21)

As seen, the impedance has a series resonance, i.e. a zero impedance frequency, for

\[
\omega = \sqrt{\frac{1}{LC}}.
\]

(3.22)

For currents with this frequency the impedance through the passive filter approaches zero, and the filter will ideally draw all currents of this frequency from the network without distortion. When tuned to the most significant harmonic frequency, the voltage distortion will thereby be reduced.

If there are several harmonics with significant value, it might be used several parallel connected filters, each tuned for one harmonic frequency.

Connecting the filter to the network, its impedance will come in parallel to the generator impedance. The resulting network impedance will then be a paralleling of the generator impedance and filter impedance

\[
Z(\omega) = Z_{\text{gen}}(\omega) | Z_{\text{filter}}(\omega) = \frac{j\omega L_g j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)}{j\omega L_g + j\omega L \left(1 - \frac{1}{\omega^2 LC}\right)}
\]

(3.23)

In addition to the series resonance with a zero impedance at \( \omega = \sqrt{\frac{1}{LC}} \), this also have a parallel resonance at \( \omega = \sqrt{\frac{1}{(1 - \omega^2 LC) L_L}} \), meaning that the impedance of harmonics with this
frequency approaches infinite high values. If this network is injected by harmonic currents of this particular frequency, the result can be excessive harmonic distortion and deterioration of equipment. Parallel resonance will always occur when passive filters are applied, the objective is to ensure that it will not be excited by any harmonic current.

In practice, both the series and the parallel resonance approach finite values due to damping effects from resistive components in the network impedance. Figure 3.66 (b) shows a typical impedance curve for a network with generators and a passive filter tuned to the 5\textsuperscript{th} harmonic. As seen, the zero impedance coincides with the 5\textsuperscript{th} harmonic, but also have some reducing effects on the 7\textsuperscript{th} and higher harmonics due to its paralleling of the inductance with the generator.

Also, it is seen that a series resonance occur at about 3\textsuperscript{rd} harmonic. This may cause problems if one expects that the network is subject to third harmonic currents, e.g. from transformer inrush. This resonance frequency can be shifted by adding a 3\textsuperscript{rd} harmonic filter to the network in parallel to the 5\textsuperscript{th} harmonic network as shown in Figure 3.67, with resulting frequency as shown.

Passive filters may be an efficient way to reduce harmonic distortion. The design may be difficult, especially if the network is complicated, in the meaning of many possible configuration alternatives. Paralleling of filter units, minimum and maximum generator configurations, maximum capacitive loading of the generators, etc. are important aspects in the design. Also, adding filters to the network will alter the load current wave forms of the converters, and the filter design will always be an iterative approach before finding the final design.

Active filters

An active filter is a power electronic unit connected to the power distribution with switching components, such as IGBTs, similar to the inverter stage of a motor drive. This feeds a capacitor bank, as shown in Figure 3.68.

By use of the switching elements, one can define a shape of the currents which shall flow from the active filter towards the network. If measuring the load current of a nonlinear load, e.g. a motor drive, the active filter can then be used to compensate for the harmonics of the nonlinear load, such that the resulting current of the nonlinear load and the active filter becomes
Figure 3.68: An active filter connected to a six-pulse converter, generating the harmonic currents that compensates for all harmonics of the non-linear converter load. The resulting current flowing to the network and generator ideally becomes sinusoidal.

sinusoidal. Due to the switching of the power semiconductors, a high frequency filter is necessary to remove high frequent noise.

Active filtering is an efficient way of removing harmonic distortion. However, the rating of the filter is relatively high compared to the nonlinear load it is supposed to filter, and the cost tend to be higher than many other alternatives.

3.8 Example Configurations

Example configurations are found in figures:

- 3.69: Offshore supply/Anchor handling vessel.
- 3.70: Ferry.
- 3.71: Shuttle tanker. The configuration shown here has been installed on several shuttle tankers and multipurpose shuttle tankers operating in accordance to requirements of the North Sea, delivered in mid 90’s. Today, a solution with podded propulsion like in Fig. 3.16 would be more likely to build.
- 3.72: Semi-submersible drilling rig.
- 3.73: Floating production vessel.
Figure 3.69: Example of single line diagram of a two-split power plant for an offshore supply vessel with four diesel-generator sets, two Azipods, one azimuthing thruster and one tunnel thruster.

Figure 3.70: Example of a single line diagram for a ferry.
Figure 3.71: Example of a single line diagram of a two-split power plant for a shuttle tanker with four diesel-generator sets, one main propulsion unit, two azimuthing thrusters and two tunnel thrusters.
Figure 3.72: Example of single line diagram of a four-split power plant for a drilling rig with eight diesel-generator sets and eight azimuthing thrusters.
Figure 3.73: Example of a single line diagram of a two-split power plant for a FPSO with five diesel-generator sets and three azimuthing thrusters.
Chapter 4

Computer-Controlled Systems

4.1 Introduction

Today most of the control systems are implemented in computers run by digital processors. For the applications studied here, computers will operate on continuous-time processes at discrete-time sampling instants. Therefore, computer-controlled systems can be regarded as discrete control systems. Discrete systems could in many cases be viewed as approximations of analog systems. However, it is important to be aware of that computer-controlled systems may also open up new possibilities that were not possible in analog systems. On the other hand computer-controlled systems may also introduce deteriorations of the control objective, if not accounting for the fact that they operate on discrete instants of processes which are time-continuous by nature. Some important aspects to consider in computer-controlled systems are:

- Signal sampling and reconstruction of time-continuous signals. In computer based control systems discrete-time approximations of continuous signals have to be done before they can be processed by the computer. The discrete-time approximation is based on the division of the time axis into sufficient small time increments. The effect of the discrete-time approximations on the control system design will be discussed.

- Signal detection and quality checking and handling of multiple signals Each sensor signal has to be checked for errors subject to certain criteria before processed by the control system. If several sensors provide measurements of the same state variable, weighting and voting mechanism must be introduced. Signal detection and handling of multiple signals will be treated in Chapter 5.

This chapter will present the basic properties of linear system theory and computer-controlled systems. Most of the theory presented is general and valid for almost all kind of industrial control problems.

The fundamental theory presented in this chapter is a short extract from well recognized text books in this field. For more thoroughly presentation the reader is referred to Johansson [138], Egeland [63], Electronics Engineer’s Handbook [61], Oppenheim et al. [215], The Control Handbook [170], Åström and Wittenmark [333], Dorf and Bishop [59], Balchen et al. [21] and Chen [47] and [48].

The class of systems studied in this text will have input and output terminals as shown in Figure 4.1.
It is assumed that the relation between excitation/cause and response/effect can be formulated by either single-input single-output (SISO or monovariable) systems or by multi-input multi-output (MIMO or multivariable) systems. The input and output signals of monovariable systems are denoted by $u(t)$ and $y(t)$, while multivariable systems will be defined by the input and output vectors $\mathbf{u}(t)$ and $\mathbf{y}(t)$. A system is called a continuous-time system, if it accepts continuous-time signals as its input and generates continuous-time signals as its output. Similarly, a system is a discrete-time system, if the input and output relation is described by discrete inputs and outputs.

Today computer-controlled systems as shown in Figure 4.2 are widely used in many industrial control applications including marine control systems. Sensor measurements are often provided by digital signals in form of serial signals and are thereby discrete. Analog (continuous-time) measurements will be made discrete by a so-called analog-to-digital (AD) converter before processed by the computer. The calculated outputs of the controller will be made analog by a digital-to-analog (D/A) converter before acting the process plant. A computer-controlled system consists of:

- The real process denoted as process plant.
- Sampler with analog-to-digital (AD) converter is converting the analog output signal of the real process into finite digital numbers. The resolution is dependent on how many bits that are used. The sampling process is quantized by the clock.
- Digital-to-analog (DA) converter with hold circuit. In Figure 4.2 a DA converter with Zero-Order-Hold (ZOH) circuit is used to convert and translate the digital signal into a continuous-time signal that is applied to the real process.
- Computer with clock and software (SW) for real-time control applications and interrupt handling. The control algorithms are implemented in a SW program and work with input data that are quantized in time and in level.
• Communication network. One should notice that in a distributed automation system as shown in Chapter 2 there are several computers with clocks that need to be synchronized with hard constraints achieving a real-time control system.

• Operator station with graphical user interface to the operator.

The process of converting a sequence of numbers into a continuous-time signal is called signal reconstruction. Hence, the calculations in the computer will be on the discretized signals. The discrete systems treated here will be discretized version of physically continuous systems as shown in the Figure 4.2. Sampling means that a continuous-time signal is replaced by a sequence of numbers representing the signal at certain times. Let \( Z = \ldots, -2, -1, 0, 1, 2, \ldots \) define the positive and negative integers, and let \( \{ t_k : k \in Z \} \) be a subset of the real numbers called the sampling instants. Then, the sampled version of the signal \( y(t) \) is the sequence

\[
\{ y(t_k) : k \in Z \}.
\]

For the systems studied here negative integers of \( k \) will not be considered. Sampling is a linear operation and the sampling instants are often equally spaced in time such that

\[
t_k = kT,
\]

where \( T \) is the sampling period or the sampling time. The discrete-time model between the input sequence \( \{ u(t_k) \} \) and the output sequence \( \{ y(t_k) \} \) is called a stroboscopic model since it gives the relationship between the system variables at the sampling instant only.

### 4.2 Basics in Linear System Theory

#### 4.2.1 Causality and State Definition

We will in this text only consider systems that current output depends on past and current inputs, hence, not future inputs. Then the system is said to be causal or nonanticipatory.
system. A noncausal system can predict what will be applied in the future by the fact that current output will depend on future input. This capability is not possible for physical systems, which are said to be causal.

By the definition of state vector denoted by \( x(t) \), a causal system’s output can be properly defined for inputs going back to \(-\infty\) to the current time \( t \).

**Definition 4.1** Chen [47]. The state \( x(t_o) \) of a system at time \( t_o \) is the information at \( t_o \) that, together with the input \( u(t) \), for \( t \geq t_o \), determines uniquely the output \( y(t) \) for all \( t \geq t_o \).

By definition, by knowing the state \( x(t) \) at \( t_o \) there is no need to know the input \( u(t) \) applied before \( t_o \) in determining the output \( y(t) \) after \( t_o \). The state summarize the effect of the past input on the future output or represents the memory that the dynamic system has of its past. Furthermore, a dynamic system is said to be lumped, if the state vector has finite number of state variables. A dynamic system is called a distributed system, if its state vector has infinitely many state variables.

### 4.2.2 Continuous-time State Space Model

A general dynamic continuous-time nonlinear process plant may be described by the state space model consisting of a finite number \( n \) of coupled first-order differential equations according to

\[
\dot{x} = f(t, x, u, w), \quad (4.3) \\
y = h(t, x, u, v), \quad (4.4)
\]

where \( \dot{x} \) denotes the time derivatives of \( x \) with respect to the time variable \( t \). The functions \( f \in \mathbb{R}^n \) and \( h \in \mathbb{R}^q \) are assumed to be smooth. The states variables are defined by the \( n \)-dimensional vector \( x \), \( u \) is the control input vector of dimension \( p \), \( w \) is the \( r \)-dimensional process disturbances vector, \( y \) the output vector of dimension \( q \) comprises variables of particular interest in the analyses of the dynamic system, and \( v \) is the \( q \)-dimensional noise vector. Here, the output vector \( y \) is regarded as physically measured variables.

Eq. (4.3) and (4.4) may be further linearized about the vector (operation points) \( x_o \). Then, for linear time-variant (LTV) systems (4.3) and (4.4) takes the form

\[
\dot{x} = A(t)x + B(t)u + E(t)w, \quad (4.5) \\
y = C(t)x + D(t)u + v, \quad (4.6)
\]

where \( A = \frac{\partial f}{\partial x} |_{x_o} \) is the \( n \times n \) dimensional system matrix, \( B = \frac{\partial f}{\partial u} |_{x_o} \) is the \( n \times p \) dimensional input matrix, \( E = \frac{\partial f}{\partial w} |_{x_o} \) is the \( n \times r \) dimensional disturbance matrix, \( C = \frac{\partial h}{\partial x} |_{x_o} \) is the \( q \times n \) dimensional measurement matrix, and \( D = \frac{\partial h}{\partial u} |_{x_o} \) is an \( q \times r \) dimensional matrix. If these matrices have constant coefficients, the system described in (4.5) and (4.6) is defined to be linear time-invariant (LTI) and will be written

\[
\dot{x} = Ax + Bu + Ew, \quad (4.7) \\
y = Cx + Du + v. \quad (4.8)
\]

Consider the simplified LTI system where process disturbance and sensor noise are disregarded

\[
\dot{x} = Ax + Bu, \quad (4.9) \\
y = Cx + Du. \quad (4.10)
\]
From linear control theory it is well known that the solution of (4.9) is

\[
x(t) = e^{A(t-t_0)}x(t_0) + \int_{t_0}^{t} e^{A(t-\tau)}Bu(\tau)d\tau.
\] (4.11)

Let us define the transition matrix to be \( \Phi(t) = e^{At} \). Furthermore, the transition matrix can be represented by a Taylor series according to

\[
\Phi(t) = e^{At} = I + At + \frac{1}{2!} A^2t^2 + \ldots + \frac{1}{\tau!} A^\tau t^\tau + \ldots
\] (4.12)

This representation will be used later in the text when making an approximated time-discrete formulation of (4.7) and (4.8).

**Definition 4.2** (Chen [47]). The state equations (4.9) and (4.10) are said to be observable if for any unknown initial state \( x(0) \), there exists a finite time \( t_1 > 0 \) such that the knowledge of the input \( u \) and the output \( y \) over \([0, t_1]\) suffices to determine uniquely the initial state \( x(0) \). Otherwise (4.9) and (4.10) are said to be unobservable.

**Theorem 4.1** (Chen [47]) The \( n \)-dimensional pair \((A, C)\) is observable if the \( nq \times n \) observability matrix \( Q_o \) has full column rank \( n \), where

\[
Q_o = [C^T, A^TC^T, \ldots, (A^T)^{n-1}C^T]^T.
\] (4.13)

**Definition 4.3** (Chen [47]). The state equation (4.9) or the pair \((A, B)\) is said to be controllable if for any initial state \( x(0) = x_0 \) and any final state \( x_1 \), there exists an input that transfers \( x_0 \) to \( x_1 \) in a finite time. Otherwise (4.9) or the pair \((A, B)\) is said to be uncontrollable.

**Theorem 4.2** (Chen [47]) The \( n \)-dimensional pair \((A, B)\) is controllable if the \( n \times np \) controllability matrix \( Q_c \) has full row rank \( n \), where

\[
Q_c = [B, AB, \ldots, (A)^{n-1}B].
\] (4.14)

**Remark 4.1** In Matlab the controllability and observability matrices can be generated by calling `ctrb` and `obsv` respectively.

**Remark 4.2** A sufficient and necessary condition for that the \( n \times n \) matrix \( Q \) has full rank is that \( Q^{-1} \) exists.

**Remark 4.3** When the vector \( x \) consists of \( n \) real numbers, it is said to be \( n \)-dimensional. Mathematically we can also write \( x \in \mathbb{R}^n \). When the matrix \( A \) consists of \( n \) rows and \( n \) columns of elements of real numbers, it is said to be \( n \times n \) dimensional, or simply \( n \times n \). Mathematically we can write \( A \in \mathbb{R}^{n \times n} \). In this text both notations will be used.

**Example 4.1** Consider the state equations

\[
\dot{x} = \begin{bmatrix} 2 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 2 & -1 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} u,
\]

\[
y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} x.
\]

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Controllability and observability can be evaluated by the following Matlab script:

```
A=[2 1 0; 2 0 1; 0 2 -1]; B=[0 1; 1 0; 0 0]; C=[1 0 1];
CO=CTRB(A,B); OB=OBSV(A,C);
rank(CO); rank(OB);
```

The state equations are both observable and controllable as Matlab returns:

$$
\begin{align*}
CO &= \begin{bmatrix} 0 & 1 & 1 & 2 & 2 & 6 \\ 1 & 0 & 0 & 2 & 4 & 4 \\ 0 & 0 & 2 & 0 & -2 & 4 \end{bmatrix},
OB &= \begin{bmatrix} 1 & 0 & 1 \\ 2 & 3 & -1 \\ 10 & 0 & 4 \end{bmatrix},
\end{align*}
$$

rank(CO) = 3, rank(OB) = 3.

4.2.3 Basic Functions

We will here define some of the most used basic functions applied in control system design and analyses.

**Step function**

Consider the sequence of functions \( \{z_n(t)\} \), where

\[
z_n(t) = \begin{cases} 
0, & t < -\frac{1}{2n} \\
nt + 1/2, & -\frac{1}{2n} \leq t \leq \frac{1}{2n} \\
1, & \frac{1}{2n} < t
\end{cases}
\]

(4.15)

where \( n \) is an integer. As seen in Figure 4.3 for \( n \to \infty \) the sequence \( \{z_n(t)\} \) converges to the **step function**, also called **Heaviside function**, by

\[
f(t) = \begin{cases} 
0, & t < 0 \\
1/2, & t = 0 \\
1, & t > 0
\end{cases}
\]

(4.16)
Impulse function

The impulse function can be thought as the derivative of the step function such that

\[ s_n = \frac{dz_n(t)}{dt} \xrightarrow{n \to \infty} \delta(t). \] (4.17)

The unit impulse \( \delta(t) \), see Figure 4.4, also known as the Dirac function or delta function is defined to have zero duration and infinite amplitude. The Dirac function is assumed to have the property

\[ \int_{t=0}^{t=0^+} \delta(t)dt = 1. \] (4.18)

The Dirac impulse function is a mathematical abstraction which cannot be realized in practice since no physical signal can have infinite amplitude with zero duration.

Ramp and parabolic functions

The unity ramp function, see Figure 4.5, can be regarded as the integral of the step function

\[ r(t) = \begin{cases} \int_t^\infty f(t)dt = t, & t > 0 \\ 0, & t \leq 0 \end{cases}. \] (4.19)

The unity parabolic function, see Figure 4.5, can be regarded as the integral of the ramp function according to

\[ p(t) = \begin{cases} \int_{t=\infty}^t r(t)dt = t^2, & t > 0 \\ 0, & t \leq 0 \end{cases}. \] (4.20)

4.2.4 Laplace Transform

The two-sided Laplace transformation \( F(s) \) of a continuous signal \( f(t) \) defined for all time \( t \) is defined as

\[ F(s) = \mathcal{L}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-st}dt, \] (4.21)
Figure 4.5: (a) Ramp function and (b) parabolic function.

where the argument $s = \sigma + j\omega$ is the complex frequency. The inverse Laplace transform is defined to be

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \frac{1}{2\pi j} \int_{\sigma-j\infty}^{\sigma+j\infty} F(s)e^{st}ds. \quad (4.22)$$

For the case $f(t)$ only takes values for $t > 0$, it is customary to restrict the Laplace transform to the be one-sided according to

$$\mathcal{L}\{f(t)\} = F(s) = \int_{0}^{\infty} f(t)e^{-st}dt. \quad (4.23)$$

The one-sided Laplace transform will be identical to the two-sided only when $f(t) = 0$ for $t \leq 0$.

In control problems we normally deal with one-sided Laplace transforms.

Table of the most used Laplace transforms are found in Dorf and Bishop [59], Balchen et al. [21] or in mathematical text books covering Laplace transforms.

4.2.5 Fourier Transform

A spectrum is defined as the Fourier transform of a signal $f(t)$ and is given

$$F(j\omega) = \mathcal{F}\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt. \quad (4.24)$$

The inverse Fourier transform is defined

$$f(t) = \mathcal{F}^{-1}\{F(j\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega)e^{j\omega t}d\omega. \quad (4.25)$$

Remark 4.4 Assume that the Fourier and Laplace transform exists, that is, take finite values (remain bounded), then the Fourier transform and the Laplace transform coincide for the choice $s = j\omega$, where $\omega$ can be regarded as the angular frequency $[\text{rad/s}]$. The major difference between the Laplace and Fourier transforms is that the Laplace transform is valuable for analysis of transient behavior, whereas the Fourier transform is mainly applicable to periodic signals.

Remark 4.5 Alternatively $s \triangleq \frac{d}{dt}, s^2 \triangleq \frac{d^2}{dt^2}, s^3 \triangleq \frac{d^3}{dt^3}, \ldots$, could also be considered to be the time differential operator which is convenient to apply in manipulation of linear differential equations with constant coefficients. Notice that an operator should not be confused with the complex variable in the Laplace transformation theory.
4.3 Sampler and Zero-Order Hold

A continuous-time signal may be discretized by a sampler. A sampler is basically a switch that closes every $T$ second for one instant of time. Let $r(t)$ be a continuous signal, where $r(t) = 0$ for $t < 0$. Let the input to the sampler be $r(t)$, then the output of the sampler is denoted $r^*(t)$. We define $kT$ to be the current sample time, where the current value of $r^*(t)$ is $r(kT)$. By introducing a periodic impulse train $p_T(t)$ and multiply it with the continuous signal, the series of impulses can be represented by

$$r^*(t) = r(t)p_T(t),$$

where $p_T(t)$ is often referred to as the sampling function and is defined to be

$$p_T(t) = \sum_{k=0}^{\infty} \delta(t - kT), \quad t > 0.$$ 

$\delta(t - kT)$ is the Dirac impulse function at $t - kT$. This mechanism is also known as impulse-train sampling.

**Remark 4.6** Notice that multiplying $r(t)$ by a unit impulse samples the value of the signal at the point at which the impulse is located. Hence, $r(t)\delta(t - kT) = r(kT)\delta(t - kT)$.

**Remark 4.7** $T$ is denoted as the sampling period and $\omega_s = \frac{2\pi}{T}$ is the fundamental frequency of $p_T(t)$, denoted as the sampling frequency in rad/s. $f_s = \frac{1}{T}$ is the sampling frequency in Hz.

We can now rewrite (4.26) such that

$$r^*(t) = r(kT)\sum_{k=0}^{\infty} \delta(t - kT).$$

In control application it is required that the sampled signal will be kept constant over the sampling interval. This may be done by a ZOH circuit in the digital-to-analog (D/A) converter. The D/A converter is often installed on separate Input/Output (I/O) boards. Thus a sampler together with a ZOH is introduced, see Figure 4.6. This will convert the sampled signal $r^*(t)$ to a continuous signal $p(t)$.

The transfer function of the ZOH circuit is given by

$$G_o(s) = \frac{1}{s} - \frac{1}{s}e^{-Ts} = \frac{(1 - e^{-sT})}{s},$$

where $s$ is the complex variable known from Laplace transformation theory. The ZOH circuit holds the value of $r(kT)$ constant for $kT \leq t < (k+1)T$. In Figure 4.6 (b) this is shown for the case the impulse input $r(kT) = 1$ for $k = 0$ and $r(kT) = 0$ for $k \neq 0$, so that $r^*(t) = r(0)\delta(t)$.

4.4 Discrete-time State Space Model

4.4.1 ZOH Equivalent of Continuous-time State Space Model

Let us define the sampling instants, $t_k = kT$, as the times when the control input $u \in \mathbb{R}^p$ changes. The zero-order-hold will result in a discontinuous control signal. We will adopt the convention
that the control signal is kept constant in the time interval \( t_k \leq t \leq t_{k+1} \) and continuous from the right. The state \( x \in \mathbb{R}^n \) at time \( t \) of (4.9) is thus given by (Åström and Wittenmark [333])

\[
x(t) = e^{A(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A(t-s')}Bu(s')ds',
\]

\[
= e^{A(t-t_k)}x(t_k) + \int_{t_k}^{t} e^{A(t-s')}ds'Bu(t_k),
\]

\[
= e^{A(t-t_k)}x(t_k) + \int_{0}^{t-t_k} e^{As}dsBu(t_k), \quad s = t - s'
\]

\[
= \Phi(t, t_k)x(t_k) + \Delta(t, t_k)u(t_k).
\]

The state vector \( x(t) \) at time \( t \) is thus a linear function of \( x(t_k) \) and \( u(t_k) \). If the AD and DA converters are perfectly synchronized and if the conversion times are negligible, the input \( u(t_k) \) and output \( y(t_k) \in \mathbb{R}^q \) can be regarded as being at the same instants such that the system equation of the sampled system at the sampling instants is given by the linear difference equation

\[
x(t_{k+1}) = \Phi(t_{k+1}, t_k)x(t_k) + \Delta(t_{k+1}, t_k)u(t_k), \quad (4.31)
\]

\[
y(t_k) = Cx(t_k) + Du(t_k), \quad (4.32)
\]

where

\[
\Phi(t_{k+1}, t_k) = e^{A(t_{k+1}-t_k)}, \quad (4.33)
\]

\[
\Delta(t_{k+1}, t_k) = \int_{t_{k+1}-t_k}^{t_{k+1}} e^{As}dsB, \quad (4.34)
\]
and $\Phi \in \mathbb{R}^{n \times n}$, $\Delta \in \mathbb{R}^{n \times p}$, $C \in \mathbb{R}^{q \times n}$ and $D \in \mathbb{R}^{q \times p}$. The zero-order-hold solution (4.31) and (4.32) gives the exact values of the state variables because the control input is constant between the sampling instants. The intersample behavior of the system, that is the state vector between the sampling instants, can be investigated by using (4.30). For periodic sampling interval, $t_k = kT$ where $k = 0, 1, 2, 3, \ldots$, the model described by (4.31) and (4.32) simplifies to a linear time-invariant system according to

$$x[(k + 1)T] = \Phi x[kT] + \Delta u[kT], \quad (4.35)$$
$$y[kT] = Cx[kT] + Du[kT], \quad (4.36)$$

where

$$\Phi = e^{AT}, \quad (4.37)$$
$$\Delta = \int_0^T e^{As}dsB. \quad (4.38)$$

### 4.4.2 Discrete-time Approximation

A discrete-time approximation based on Euler’s method for integration of the state space equations may be found by approximating the derivative with a difference

$$\dot{x} = \frac{x(t_k + T) - x(t_k)}{T}. \quad (4.39)$$

As above, let $k$ be an integer index so that $k = 0, 1, 2, 3, \ldots$. If we assume small sampling periods, a discrete-time approximation of (4.7) may be written

$$x(t + T) \approx (TA + I)x(t) + TBu(t) + TEw(t). \quad (4.40)$$

Introducing discrete notation (4.40) is written as

$$x[(k + 1)T] \approx (I + TA)x[kT] + TBu[kT] + TEw[kT]. \quad (4.41)$$

We can now recognize that $(I + TA)$ is the same as the first two terms of (4.12). If we assume small $T$, the transition matrix can be approximated by $\Phi(T) \approx (I + TA)$. Similarly, the control input and disturbance matrices become $\Delta = TB$ and $\Gamma = TE$. By omitting $T$ in the arguments such that $x(t) = x(kT) \approx x[k]$, (4.41) can be rewritten

$$x[k + 1] = \Phi x[k] + \Delta u[k] + \Gamma w[k], \quad (4.42)$$

and the corresponding discrete-time formulation of (4.8) becomes

$$y[k] = Cx[k] + Du[k] + v[k]. \quad (4.43)$$

**Remark 4.8** The control input matrix is often written

$$\Delta = A^{-1}(e^{AT} - I)B. \quad (4.44)$$
Remark 4.9 In several textbooks and Matlab, the discrete-time state space formulation is written using the same matrix symbols as for time-continuous systems. This means that for LTI systems (4.42) and (4.43) are reformulated according to

\[ x[k+1] = Ax[k] + Bu[k] + Ew[k], \]
\[ y[k] = Cx[k] + Du[k] + v[k]. \]  

The corresponding discrete-time version for a LTV system is given by

\[ x[k+1] = A[k]x[k] + B[k]u[k] + E[k]w[k], \]
\[ y[k] = C[k]x[k] + D[k]u[k] + v[k]. \]

4.4.3 Solution of Discrete-time System Equation

Consider the following difference equation

\[ x[k+1] = \Phi x[k] + \Delta u[k], \]
\[ y[k] = Cx[k]. \]  

A solution of (4.49) can be found by assuming that the initial condition \( x[k_0] \) and the input sequence \( u[k], u[k_0 + 1], \ldots \) are known. Thus by iteration we have

\[ x[k_0 + 1] = \Phi x[k_0] + \Delta u[k_0], \]
\[ x[k_0 + 2] = \Phi x[k_0 + 1] + \Delta u[k_0 + 1], \]
\[ = \Phi^2 x[k_0] + \Phi \Delta u[k_0] + \Delta u[k_0 + 1], \]
\[ x[k_0 + 3] = \Phi x[k_0 + 2] + \Delta u[k_0 + 2], \]
\[ = \Phi (\Phi^2 x[k_0] + \Phi \Delta u[k_0] + \Delta u[k_0 + 1]) + \Delta u[k_0 + 2], \]
\[ = \Phi^3 x[k_0] + \Phi^2 \Delta u[k_0] + \Phi \Delta u[k_0 + 1] + \Delta u[k_0 + 2], \]
\[ \vdots \]
\[ x[k_0 + k'] = \Phi^{k'} x[k_0] + \sum_{i=0}^{k'-1} \Phi^{k'-i-1} \Delta u[i + k_0]. \]  

Let us define \( k = k_0 + k' \) and \( j = i + k_0 \), then (4.51) can be reformulated

\[ x[k] = \Phi^{k-k_0} x[k_0] + \sum_{i=0}^{k-k_0-1} \Phi^{k-(i+k_0)-1} \Delta u[i + k_0], \]
\[ x[k] = \Phi^{k-k_0} x[k_0] + \sum_{j=k_0}^{k-1} \Phi^{k-j-1} \Delta u[j]. \]  

We can observe that the first part depends on the initial condition, while the second part is a weighted sum of the input signals. It is also seen that the eigenvalues of \( \Phi \) obtained from the characteristic equation

\[ \det(\lambda I - \Phi) = 0. \]  

will determine the properties of the solution (4.52).
Theorem 4.3 Asymptotic stability of linear systems (Åstrøm and Wittenmark [333]). A discrete-time linear time-invariant system on the form $x[k+1] = \Phi x[k]$ is asymptotically stable if and only if all eigenvalues of $\Phi$ are strictly inside the unit disk, that is $|\lambda_i| < 1$ for $i = 1, 2, \ldots, n$.

Remark 4.10 In general for discrete-time systems the stability is defined for a particular solution and not for the system, and is thus a local concept. However, for linear time invariant discrete-time systems, stability is also a property of the system.

4.4.4 Controllability and Observability of Discrete-time Systems

Let us define $k_0 = 0$ and $k = n$ in (4.52) such that

$$x[n] = \Phi^n x[0] + \sum_{j=0}^{n-1} \Phi^{n-j-1} \Delta u[j],$$

$$= \Phi^n x[0] + \Phi^{n-1} \Delta u[0] + \Phi^{n-2} \Delta u[1] + \Phi^{n-3} \Delta u[2] + \ldots + \Delta u[n-1],$$

$$= \Phi^n x[0] + Q_{dc} U,$$  

(4.54)

where

$$Q_{dc} = [\Delta, \Phi \Delta, \ldots, (\Phi)^{n-1}\Delta],$$

(4.55)

$$U = [u^T[n-1], \ldots, u^T[0]]^T.$$  

(4.56)

If $Q_{dc}$ has full row rank $n$, it is possible to find $n$ equations from which the control signal can be found such that the initial state is transferred to the desired final state $x[n]$.

Definition 4.4 (Chen [47]) The discrete LTI equations (4.49) and (4.50) with $n \times n$ state $\Phi$ and $n \times p$ control $\Delta$ matrices described by the pair $(\Phi, \Delta)$ are defined to be be controllable if for any initial state $x[0] = x_0$ and any final state $x[n] = x_n$, there exists an input sequence of finite length that transfers $x[0]$ to $x[n]$. Otherwise the discrete LTI equations (4.49) and (4.50) or the pair $(\Phi, \Delta)$ are said to be uncontrollable.

Theorem 4.4 (Chen [47]) The $n$-dimensional pair $(\Phi, \Delta)$ is controllable if the $n \times np$ controllability matrix $Q_{dc}$ has full row rank $n$, where

$$Q_{dc} = [\Delta, \Phi \Delta, \ldots, (\Phi)^{n-1}\Delta].$$

(4.57)

Chen [47] defines three different controllability definitions:

1. Transfer any state to any other state in finite time, as adopted here.
2. Transfer any state to the zero state in finite time, called controllability to the origin.
3. Transfer the zero state to any state in finite time, called controllability from the origin or, more often, reachability.

Remark 4.11 In the literature, there are different definitions of controllability and reachability. In Åstrøm and Wittenmark [333] definition 1 is called reachability, while definition 2 is called controllability.
Remark 4.12 Controllability definitions 1 and 2 are equivalent if $\Phi$ is invertible. Definition 2 does not imply definition 1. If $\Phi^n x[0] = 0$, then the origin of (4.54) will be reached with zero input, but the system is not necessarily controllable.

Consider (4.49) and (4.50), and for simplicity assume that the control input is equal to zero, $u[k] = 0$ for all $k$. This gives the following set of equations

$$y[0] = Cx[0],$$
$$y[1] = Cx[1] = C\Phi x[0],$$
$$\vdots$$
$$y[n-1] = C\Phi^{n-1} x[0],$$

or on vector form

$$\begin{bmatrix} C^T, (C\Phi)^T, \ldots, (C\Phi^{n-1})^T \end{bmatrix}^T x[0] = \begin{bmatrix} y^T[0], y^T[1], \ldots, y^T[n-1] \end{bmatrix}^T,$$

$$Q_{do} x[0] = \begin{bmatrix} y^T[0], y^T[1], \ldots, y^T[n-1] \end{bmatrix}^T. \quad (4.59)$$

**Definition 4.5** (Chen [47]) The discrete LTI equations (4.49) and (4.50) with $n \times n$ state matrix $\Phi$ and $q \times n$ output matrix $C$ described by the pair $(\Phi, C)$ are defined to be observable if for any unknown initial state $x[0]$, there exists a finite integer $k_1 > 0$ such that the knowledge of the input sequence $u[k]$ and output sequence $y[k]$ from $k = 0$ to $k_1$ suffices to determine uniquely the initial state $x[0]$. Otherwise the discrete LTI equations (4.49) and (4.50) or the pair $(\Phi, C)$ are said to be unobservable.

**Theorem 4.5** (Chen [47]) The $n$-dimensional pair $(\Phi, C)$ is observable if the $nq \times n$ observability matrix $Q_{do}$ has full column rank $n$, where

$$Q_{do} = [C^T, \Phi^T C^T, \ldots, (\Phi^T)^{n-1} C^T]^T. \quad (4.60)$$

**Definition 4.6** (Åström and Wittenmark [333]). The discrete LTI equations (4.49) and (4.50) is detectable if the only unobservable states are such that they decay to the origin. That is, the corresponding eigenvalues are stable.

### 4.5 Discrete-time Approximation Methods

A solution of (4.49) was found by iteration and was given in (4.52). An advantage of this method is that stability problems due to discretization may be avoided providing accurate calculation of $\Phi$. However, the main limitation is that this method in practice is only applicable for linear systems with constant coefficients. We will here briefly present three alternative numerical integration schemes often used in control applications. Numerous of other numerical methods exist, whereof the so-called Runge Kutta methods often are favorable with respect to accuracy, see Egeland and Gravdal [64] and Cheney and Kincaid [49] for details.

We will here consider a system on the form

$$\dot{x} = f(t, x, u). \quad (4.61)$$
4.5.1 Euler’s Method

The Euler’s method is based on the assumption that the derivative is approximated by the forward difference according to

\[ \dot{x} = \frac{x(t + T) - x(t)}{T} = f(t, x, u). \]  

(4.62)

We introduce the discrete notation \( t = kT \), then the integration method is written

\[ x[k + 1] = x[k] + T f(x[k], u[k], k). \]  

(4.63)

Euler’s method is an explicit 1-step method. An explicit difference equation means that \( x[k + n] \) can be recursively calculated, since all terms on the right hand side are calculated in previous sampling instants, that is for \( k+n-1, k+n-2, \ldots \). The method is denoted to be 1-step, since only values of the previous sampling is included. This method has local interrupt error proportional to \( T^2 \), often written \( O(T^2) \). Euler’s method is suitable in simulation of simple systems, where the requirements to the accuracy is not strict. The accuracy is of course sensitive to the sampling interval. It can be shown that Euler’s method is stable for systems with its poles \( \lambda_p \) within the circle \( |1 + T\lambda_p| = 1 \). For systems with real poles in the left-hand plane, a requirement for stability is

\[ T < \frac{2}{|\lambda_{\text{max}}|}. \]  

(4.64)

4.5.2 Backward Euler’s Method

The backward Euler’s method is based on the assumption that the derivative is approximated by the backward difference according to

\[ \dot{x} = \frac{x(t) - x(t - T)}{T} = f(t, x, u). \]  

(4.65)

Using discrete notation

\[ x[k] = x[k - 1] + T f(x[k], u[k], k), \]  

(4.66)

or

\[ x[k + 1] = x[k] + T f(x[k + 1], u[k + 1], k). \]  

(4.67)

Also this method is having local interrupt error of order \( O(T^2) \). Euler’s method is an implicit 1-step method. An implicit difference equation means that \( x[k + n] \) cannot be recursively calculated, since terms on the right hand side are also given at the time instant \( k + n \). The region of stability for backward Euler’s method is

\[ \frac{1}{|1 - T\lambda_p|} < 1. \]

4.5.3 Combined Backward and Forward Euler’s Method

An appropriate method for integration of second order systems with complex conjugated poles is the combined backward and forward Euler’s method. Consider the following second order system on phase-variable form

\[ \dot{x}_1 = x_2, \]  

(4.68)

\[ \dot{x}_2 = f(t, x, u). \]  

(4.69)
Let us apply Euler’s method to integrate (4.69) and backwards Euler’s method on (4.68) according to

\[ x_2[k+1] = x_2[k] + Tf(x_1[k], x_2[k], u[k], k), \] (4.70)
\[ x_1[k+1] = x_1[k] + T x_2[k+1]. \] (4.71)

We avoid the problem with the implicit equation (4.71) since of \( x_2[k+1] \) is known from the previous calculation given in (4.70). Hence, we can solve the integration recursively.

### 4.5.4 Trapezoidal (Tustin’s) Method

The trapezoidal method is an implicit 2-step method and is written

\[ x[k+1] = x[k] + \frac{T}{2} [f(x[k], u[k], k) + f(x[k+1], u[k+1], k+1)]. \] (4.72)

Condition for stability is

\[ \left| \frac{1 + \frac{T \lambda_p}{2}}{1 - \frac{T \lambda_p}{2}} \right| < 1, \quad p = 1, \ldots, n \] (4.73)

where \( \lambda_p \) is the poles of the system. If \( \text{Re}(\lambda_p) < 0 \), then \( \text{Re}(T \lambda_p) < 0 \) provided \( T > 0 \). The trapezoidal method is stable for all \( T > 0 \), and is having local interrupt error of order \( O(T^3) \).

### 4.5.5 Second Order System

In the following examples we will show how Tustin’s method subject to smaller sampling periods converge to the real solution, and then a comparison is done for backward Eulers, forward Euler’s and Tustin’s methods for discretization. Let us consider the following second order system

\[ \ddot{x} + 2 \zeta \omega_0 \dot{x} + \omega_0^2 x = \frac{1}{m} F, \] (4.74)

where \( m=10 \text{ kg} \), \( \zeta=0.2 \), \( \omega_0 = \frac{2 \pi}{T} = \frac{2 \pi}{2} \) and \( F=10 \text{ N} \). The eigenvalues are \( \lambda_p = -0.6283 \pm 3.0781i \).

**Example 4.2** Let us consider Tustin’s method for discretization of (4.74) with increasing sampling periods from \( T=0.01s \), \( 0.05s \), \( 0.1s \), \( 0.2s \), and \( 0.5s \). As seen in Figure 4.7 for \( T \leq 0.1s \) the system becomes well approximated. For \( T \geq 0.2s \), the approximation becomes oscillatory.

**Example 4.3** Let us compare backward Eulers, forward Euler’s and Tustin’s methods for discretization of (4.74) with \( T= 0.1s \). As seen in Figure 4.8 forward Euler is as expected less accurate.

### 4.6 Nyquist Frequency

Assume that a continuous signal is band limited, that is the Fourier transform is zero outside a finite band of frequencies. If the samples of the continuous signal are taken sufficiently close together in relation to the highest frequency of the band, then the samples will uniquely reconstruct the signal. Stable continuous LTI systems have the property that the steady-state response to
Figure 4.7: Step response of second order system using Tustin’s method with increasing sampling period.

Figure 4.8: Step response of second order system using backward Euler’s, forward Euler’s and Tustin’s methods.
sinusoidal excitations is sinusoidal with the frequency of the excitation signal (usually with some phase lag). However, discrete systems will create signals with new frequencies. A discrete control system operates on process variables at discrete times only. Achieving satisfactory performance and stability of the discrete control system, appropriate choice of the sampling period is crucial. The main concern is under what condition will the discrete value recover the continuous signal. In order to have a minimum fair representation of sinusoidal signals with frequency \( \omega \), Nyquist showed that it was necessary to sample at least twice per period, \( T < \frac{\pi}{\omega_M} \).

Consider the spectrum of the sampled signal \( r^*(t) \) obtained as the discrete Fourier transform

\[
R_d(j\omega) = \mathcal{F}\{r^*(t)\} = \mathcal{F}\{r(t)\} * \mathcal{F}\{p_T(t)\} = R(j\omega) * P(j\omega),
\]

(4.75)

where

\[
P(j\omega) = \mathcal{F}\left\{ \sum_{k=0}^{\infty} \delta(t - kT) \right\} = \frac{2\pi}{T} \sum_{k=0}^{\infty} \delta(\omega - \frac{2\pi}{T} k).
\]

(4.76)

Hence, since convolution with impulses shifts a signal, we have

\[
R_d(j\omega) = \frac{1}{T} \sum_{k=0}^{\infty} R(j(\omega - \frac{2\pi}{T} k)).
\]

(4.77)

Hence, \( R_d(j\omega) \) is a periodic function of the sampling frequency \( \omega_s \), see Figure 4.9 (a) and (b). In Figure 4.9 (c) we can see that if \( \omega_M < (\omega_s - \omega_M) \) or \( \omega_s > 2\omega_M \), there is no overlap between the shifted replicas of \( R(j\omega) \) and is thus properly recovered by \( R_d(j\omega) \). However, if the signal to be sampled contains frequency components larger than half the sampling frequency \( \omega_s < 2\omega_M \), the sampling process itself will create low frequency components in the sampled signal. Sampling of a signal with frequency \( \omega_M \) creates signal components with frequencies

\[
\omega_{\text{amp}} = n\omega_s \pm \omega_M
\]

where \( n \) is an arbitrary integer. The phenomenon that the frequency process creates new frequency components is called aliasing, see Figure 4.9 (d). The frequency that is half of the sampling frequency \( \omega_s \) is called the Nyquist frequency

\[
\omega_n = \frac{1}{2}\omega_s = \frac{\pi}{T}.
\]

(4.78)

To avoid aliasing it is important that all frequency components of signal higher than the Nyquist frequency are removed before the sampling. This could be done by introducing a presampling filter or antialiasing filter, which will be of lowpass characteristics. This is often done by an analog low pass filter in the front of the sampler. The lowpass filter must have a cut-off frequency equal or less the Nyquist frequency, \( \omega_c \leq \omega_n \). In Figure 4.10 we can see the recovery of a time-continuous signal from its samples using ideal lowpass filter.

**Theorem 4.6** Shannon’s sampling theorem. A continuous-time signal with a Fourier transform that is zero outside the interval \( (-\omega_M, \omega_M) \) is given uniquely by its value in equidistant points if the sampling frequency is higher than \( 2\omega_M \).

**Remark 4.13** The Fourier transform \( R_d(j\omega) \) of a sampled signal is a periodic function of the original spectrum \( R(j\omega) \) along the frequency axis with the period equal to the sampling frequency \( \omega_s \).
Figure 4.9: (a) Power spectrum of continuous-time signal, (b) power spectrum of sampling function, (c) power spectrum of sampled signal with $\omega_s > 2\omega_M$, and (d) power spectrum of sampled signal with $\omega_s < 2\omega_M$ resulting in aliasing.

Figure 4.10: Recovery of a time-continuous signal from its samples using ideal lowpass filter.
4.7 The $z$-Transform

4.7.1 Definition

Let us define

$$z = e^{Ts},$$  \hspace{1cm} (4.79)

$$z^{-1} = e^{-Ts}.$$  \hspace{1cm} (4.80)

The $z$-transform maps a semi-infinite time sequence (only a sequence of positive integers) into a function of a complex variable $z$. Applying Laplace transform on (4.28) we have

$$L\{r^*(t)\} = \sum_{k=0}^{\infty} r(kT)e^{-kTs}. \hspace{1cm} (4.81)$$

A new $z$-transform is then defined, so that for a general function $r(t)$ the $z$-transform is found to be

$$Z\{r(t)\} = Z\{r^*(t)\} = R(z) = \sum_{k=0}^{\infty} r(kT)z^{-k}. \hspace{1cm} (4.82)$$

Tables of the most used $z$-transforms are found in Åström and Wittenmark [333], Dorf and Bishop [59] and Balchen et al. [21].

Definition 4.7 For a discrete signal $f[k]$ a similar transformation as the Laplace transformation, which is defined as the $z$-transform $F(z)$ can be found by

$$F(z) = Z\{f[k]\} = \sum_{k=0}^{\infty} f[k]z^{-k}. \hspace{1cm} (4.83)$$

Example 4.4 Consider the exponential function

$$f(t) = e^{-at}, \hspace{1cm} t \geq 0. \hspace{1cm} (4.84)$$

The $z$-transform is determined to be

$$Z\{e^{-at}\} = F(z) = \sum_{k=0}^{\infty} e^{-akT}z^{-k} = \sum_{k=0}^{\infty} (ze^{aT})^{-k}, \hspace{1cm} (4.85)$$

which can be rewritten in closed form as

$$F(z) = \frac{1}{1 - (ze^{aT})^{-1}} = \frac{z}{z - e^{-aT}}. \hspace{1cm} (4.86)$$

Example 4.5 Consider the step function $f(t)$ given in (4.16). The $z$-transform is determined to be

$$Z\{f(t)\} = F(z) = \sum_{k=0}^{\infty} z^{-k}, \hspace{1cm} (4.87)$$

which can be rewritten in closed form as

$$F(z) = \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}. \hspace{1cm} (4.88)$$
4.7.2 Stability Properties

Consider \( z = e^{Ts} \), where \( s = \sigma + j\omega \). We then find

\[
    z = e^{Ts} = e^{(\sigma+j\omega)T},
\]

where

\[
    |z| = e^{\sigma T},
\]

\[
    \angle z = \omega T.
\]

We observe that the left-half plane in the \( s \)-domain with \( \sigma \leq 0 \) will in the \( z \)-domain correspond to

\[
    0 < |z| = e^{\sigma T} \leq 1.
\]

Hence the left-half plane in the \( s \)-domain corresponds to the unit circle in the \( z \)-domain. Stable roots in the \( s \)-domain will in the \( z \)-domain be within the unit circle, see Figure 4.11.

4.8 The Pulse-Transfer Function and the Pulse Response

4.8.1 Pulse-transfer Function

Consider the LTI system on the form

\[
    x[k + 1] = \Phi x[k] + \Delta u[k], \quad (4.93)
\]

\[
    y[k] = Cx[k] + Du[k]. \quad (4.94)
\]

The \( z \)-transform gives

\[
    \sum_{k=0}^{\infty} z^{-k} x[k + 1] = z \left( \sum_{k=0}^{\infty} z^{-k} x[k] - x[0] \right) = \sum_{k=0}^{\infty} \Phi z^{-k} x[k] + \sum_{k=0}^{\infty} \Delta z^{-k} u[k]. \quad (4.95)
\]

Hence,

\[
    z(X(z) - x[0]) = \Phi X(z) + \Delta U(z), \quad (4.96)
\]

\[
    \Downarrow
\]

\[
    X(z) = (zI - \Phi)^{-1} (z x(0) + \Delta U(z)). \quad (4.97)
\]
and

\[ Y(z) = C(zI - \Phi)^{-1}z x(0) + \left(C(zI - \Phi)^{-1} \Delta + D\right) U(z). \]  

(4.98)

The pulse-transfer function is defined as

\[ H(z) = C(zI - \Phi)^{-1} \Delta + D. \]  

(4.99)

4.8.2 Pulse Response

Let \( \delta[k-n] \) describe the discrete impulse sequence defined by

\[ \delta[k-n] \triangleq \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases}, \]  

(4.100)

where \( k \) and \( n \) are both integers. \( \delta[k-n] \) is the discrete counterpart of the Dirac function (4.17). When the input \( u[n] = \delta[0] \) acts on a LTI system, the discrete impulse response \( h[k] \) appears. When a input signal \( u[n] \) act on a LTI system with impulse response \( h[k] \), then the discrete output \( y[k] \) is defined as the discrete convolution integral according to

\[ y[k] = h[k]u[0] + h[k-1]u[1] + \ldots + h[1]u[k-1] + h[0]u[k], \]

\[ = \sum_{n=0}^{k} h[k-n]u[n] = \sum_{n=0}^{k} h[n]u[k-n] = h[k] \ast u[k]. \]

(4.101)

If we substitute (4.101) into (4.82) and let \( k \to \infty \), we can find the discrete pulse response function

\[ Y[z] = \sum_{k=0}^{\infty} \left( \sum_{n=0}^{\infty} h[k-n]u[n] \right) z^{-k} = \sum_{k=0}^{\infty} \left( \sum_{n=0}^{\infty} h[k-n]u[n] \right) z^{-(k-n)}z^{-n}, \]

\[ = \sum_{n=0}^{\infty} \left( \sum_{k=0}^{\infty} h[k-n]z^{-(k-n)} \right) u[n]z^{-n} = \left( \sum_{l=0}^{\infty} h[l]z^{-l} \right) \left( \sum_{n=0}^{\infty} u[n]z^{-n} \right), \]

\[ \triangleq H(z)U(z). \]

(4.102)

where \( l = k-n \) and \( h[l] = 0 \) for \( l < 0 \). As for continuous systems, we can see that convolution in the time domain (\( t \)-continuous/\( k \)-discrete) corresponds to multiplication in the frequency domain (\( s \)-continuous/\( z \)-discrete).

**Theorem 4.7** The pulse response (4.102) and the pulse-transfer function (4.99) are a \( z \)-transform pair, that is

\[ Z\{h[k]\} = H(z). \]

(4.103)

**Example 4.6** For a continuous SISO system

\[ \dot{x} = -ax + bu, \]

(4.104)

where \( x(t) = 0 \) for \( t \leq 0 \), the corresponding convolution integral will be

\[ y(t) = \int_{0}^{t} h(t-\tau)u(\tau)d\tau = h(t) \ast u(t), \]

(4.105)

where \( h(t-\tau) = e^{-a(t-\tau)}b. \)
Example 4.7 Consider the unit-sampling-time delay system defined by

\[ y[k] = u[k - 1]. \quad (4.106) \]

The output equals the input delayed by one sampling period. Its impulse response is \( h[k] = \delta[k-1] \) and its discrete transfer function is

\[ H(z) = Z\{\delta[k - 1]\} = z^{-1} = \frac{1}{z}. \quad (4.107) \]

One should notice that every continuous system involving time delay is a distributed system. This is not the so in the discrete case, where \( H(z) \) is a rational function in \( z \). Similarly the discrete transfer function of \( h[k] = \delta[k - m] \) is

\[ H(z) = Z\{\delta[k - m]\} = z^{-m}. \quad (4.108) \]

Remark 4.14 Notice that if \( h[k] \neq 0 \) for only a finite number of \( k \), the system is called a finite impulse-response (FIR) system. This means that the output only will be influence by a finite number of inputs.

4.9 Shift-operator Calculus

For continuous systems differential-operator calculus, where \( s \triangleq \frac{d}{dt}, s^2 \triangleq \frac{d^2}{dt^2}, s^3 \triangleq \frac{d^3}{dt^3}, \ldots \), is a helpful tool for manipulating linear differential equations with constant coefficients. For discrete systems a similar equivalent shift-operator calculus can be defined for linear difference equations with constant coefficients. All signals to be considered has to be defined by double sequences according to

\[ \{y[k] : k = \ldots -2, -1, 0, 1, 2, \ldots\}. \quad (4.109) \]

4.9.1 Shift Operator

The forward-shift operator is here denoted by \( z \) such that

\[ zu[k] = u[k + 1]. \quad (4.110) \]

Remark 4.15 Notice that in some literature such as Åström and Wittenmark [333] the letter \( q \) is used in order to have a clear distinguish from the \( z \)-transform which is a complex variable which also take initial conditions into consideration. However, here we will use \( z \) as long as it is clear from the text which purpose \( z \) is used. Forward-shift operator is often used in relation to stability analysis related to the characteristic equation.

Example 4.8 Åström and Wittenmark [333]. Consider the difference equation

\[ y[k + 1] - ay[k] = u[k], \quad (4.111) \]

where \( |a| < 1 \). In operator notation the equation can be written as

\[ (z - a)y[k] = u[k]. \quad (4.112) \]
A solution is
\[ y[k] = \frac{1}{z-a} u[k] = \frac{z^{-1}}{1-az^{-1}} u[k]. \]  
(4.113)

Since \( z^{-1} \) has unit norm, the right-hand side can be represented as a convergent series according to
\[ y[k] = z^{-1} (1 + az^{-1} + a^2 z^{-2} + \ldots) u[k], \]
\[ = \sum_{i=1}^{\infty} a^{i-1} u[k-i]. \]  
(4.114)

If \( y[k_0] = y_0 \) it follows from (4.52) that the solution is
\[ y[k] = a^{k-k_0} y_0 + \sum_{j=k_0}^{k-1} a^{k-j-1} u[j], \]
\[ = a^{k-k_0} y_0 + \sum_{i=1}^{k-k_0} a^{i-1} u[k-i]. \]  
(4.115)

It is clear that (4.114) is equal to (4.115) only if it is assumed that \( y_0 = 0 \) or that \( k - k_0 \to \infty \).

The inverse of the forward-shift operator is called the backward-shift operator or the delay operator, and is denoted by \( z^{-1} \) such that
\[ z^{-1} u[k] = u[k-1]. \]  
(4.116)

If the range is not double infinite sequences, the inverse of the forward-shift operator may not exist. The backward-shift operator is often used in relation to causality considerations.

### 4.9.2 Pulse-transfer Operator

Consider the LTI system on the form
\[ x[k+1] = \Phi x[k] + \Delta u[k], \]  
\[ y[k] = C x[k] + D u[k]. \]  
(4.117, 4.118)

An input-output relationship is obtained by
\[ z x[k] = \Phi x[k] + \Delta u[k]. \]  
(4.119)

Hence,
\[ x[k] = (zI - \Phi)^{-1} \Delta u[k], \]  
(4.120)

and
\[ y[k] = \left( C (zI - \Phi)^{-1} \Delta + D \right) u[k]. \]  
(4.121)

The pulse-transfer operator is defined as
\[ H(z) = C (zI - \Phi)^{-1} \Delta + D = \frac{B(z)}{A(z)}. \]  
(4.122)

The poles of (4.122) is given by the zeros of the characteristics polynomial \( A(z) \).
4.10 Stability Regions

Some important aspects concerning stability, when applying the different approximation methods for integration as presented in Section 4.5, appears nicely by applying the shift-operator calculus. This will be demonstrated by examples.

Example 4.9 Egeland [63]. Consider the LTI on the form
\[ \dot{x} = Ax. \] (4.123)

Let us then apply the trapezoidal method given by (4.72) according to
\[ x[k+1] = x[k] + \frac{T}{2} [Ax[k] + Ax[k+1]]. \] (4.124)

If we apply the shift-operator calculus, we have
\[ zx[k] = x[k] + \frac{T}{2} [Ax[k] + zAx[k]], \]
\[ \Downarrow \]
\[ \frac{2}{T} \left( \frac{z-1}{z+1} \right) x[k] = Ax[k]. \] (4.125)

Alternatively (4.126) can be rewritten
\[ \frac{2}{T} \left( \frac{1-\frac{1}{z}}{1+\frac{1}{z}} \right) x[k] = Ax[k]. \] (4.127)

Example 4.10 Let us then apply Euler’s method based on forward difference (4.63) on (4.123) according to
\[ x[k+1] = x[k] + TAx[k]. \] (4.128)

If we apply the shift-operator calculus, we have
\[ zx[k] = x[k] + TAx[k], \]
\[ \Downarrow \]
\[ \left( \frac{z-1}{T} \right) x[k] = Ax[k]. \] (4.130)

Example 4.11 Let us then apply Euler backward method (4.66) on (4.123) according to
\[ x[k+1] = x[k] + TAx[k+1]. \] (4.131)

If we apply the shift-operator calculus, we have
\[ zx[k] = x[k] + TzAx[k], \]
\[ \Downarrow \]
\[ \left( \frac{z-1}{zT} \right) x[k] = Ax[k]. \] (4.133)
Remark 4.16 Transformation from the $s$-domain to the $\zeta$-domain may appear by using one of the following approximations

\[ s = \frac{2(1 - z^{-1})}{T(1 + z^{-1})}, \] (4.134)

\[ s = \frac{z - 1}{T}, \] (4.135)

\[ s = \frac{z - 1}{zT}. \] (4.136)

Eq. (4.134) is known as the bilinear transformation or Tustin’s approximation often used in stability analysis of discrete controllers in the so-called $q$-domain, see Egeland [63] and Balchen et al. [21]. This transformation maps the imaginary axis in the $s$-domain to the unit circle in the $\zeta$-domain. Thus, stable poles in the left-hand plane in the $s$-domain correspond to poles within the unit circle in the $\zeta$-domain.

Example 4.12 Egeland [63]. Consider the second order system given by

\[ \dot{x}_1 = x_2, \] (4.137)

\[ \dot{x}_2 = -\omega_0^2 x_1 - 2\zeta \omega_0 x_2 + v, \] (4.138)

where $\zeta$ is the damping ratio, $\omega_0$ is the resonance frequency and $v$ is the disturbance. Applying Euler’s method of integration gives

\[ x_1[k + 1] = x_1[k] + T x_2[k], \] (4.139)

\[ x_2[k + 1] = x_2[k] + T (-\omega_0^2 x_1[k] - 2\zeta \omega_0 x_2[k] + v[k]). \] (4.140)

By using shift-operator calculus we have the $\zeta$-domain formulation

\[ \left(\frac{z - 1}{T}\right) x_1[k] = x_2[k], \] (4.141)

\[ \left(\frac{z - 1}{T}\right) x_2[k] = (-\omega_0^2 x_1[k] - 2\zeta \omega_0 x_2[k] + v[k]). \] (4.142)

Assume $\zeta = 0$, by setting the expression for $x_2[k]$ given by (4.141) into (4.142), the the $\zeta$-domain formulation appears to be

\[ \frac{x_1}{v}(z) = \frac{T^2}{z^2 - 2z + 1 + T^2 \omega_0^2}. \] (4.143)

Notice that this expression has poles for $z_{1,2} = 1 \pm jT \omega_0$. Thus, the poles is outside the unit circle, such that the discretized system is unstable for all $T$.

Example 4.13 Egeland [63]. Consider the second order system given by (4.137) and (4.138). Applying combined forward and backwards Euler’s method of integration gives

\[ x_2[k + 1] = x_2[k] + T (-\omega_0^2 x_1[k] - 2\zeta \omega_0 x_2[k] + v[k]), \] (4.144)

\[ x_1[k + 1] = x_1[k] + T x_2[k + 1]. \] (4.145)
The shift-operator calculus gives

\[ x_2[k] = \frac{T}{(z - 1)} \left( -\omega_0^2 x_1[k] - 2\zeta\omega_0 x_2[k] + v[k] \right), \quad (4.146) \]
\[ x_1[k] = \frac{Tz}{(z - 1)} r_2[k]. \quad (4.147) \]

Assume \( \zeta = 0 \), by setting (4.147) into (4.146) yields the \( z \)-domain formulation

\[ \frac{x_1}{v}(z) = \frac{z^2 - 2z + 1}{z^2 + (2\omega_0^2 - 2)z + 1}. \quad (4.148) \]

Notice that this expression has poles for \( z_{1,2} = 1 - \frac{2\omega_0^2}{2} \pm T\omega_0 \sqrt{\frac{2\omega_0^2}{4} - 1} \). The poles are complex conjugated for \( T < \frac{2}{\omega_0} \) such that

\[ z_{1,2} = 1 - \frac{T^2\omega_0^2}{2} \pm T\omega_0 \sqrt{1 - \frac{T^2\omega_0^2}{4}} j = a \pm jb, \quad (4.149) \]

where \( a = 1 - \frac{T^2\omega_0^2}{2} \) and \( b = T\omega_0 \sqrt{1 - \frac{T^2\omega_0^2}{4}} \). Since \( z_1 z_2 = a^2 + b^2 = 1 \) this indicates that the poles are on the unit circle, which corresponds to the poles on the imaginary axis in the \( s \)-domain. Hence, stability is ensured.

In Figure 4.12 it is shown how the stability region for \( \text{Re}(s) < 0 \) is mapped on the \( z \)-plane for Euler’s method (4.135), backwards Euler (4.136) and trapezoidal method (4.134). We observe that Euler’s method may map the time-continuous system into an unstable region in the discrete-time system, that is outside the unit circle. For systems with complex conjugated poles with small damping Euler’s method may cause stability problems as we noticed in the example above. When the backward approximation is used a stable continuous-time system always give a stable discrete-time system. Unfortunately, some unstable continuous-time systems may be transformed into stable discrete systems. Tustin’s approximation or the trapezoidal method has the advantage that the left-hand of the \( s \)-plane is transformed into the whole unit circle. Hence, stable (unstable) continuous-time systems are transformed into stable (unstable) discrete systems.
4.11 Order of the System

Let a difference equation be written as

$$y[k + n_a] + a_1y[k + n_a - 1] + ... + a_{n_a}y[k] = b_0u[k + n_b] + b_1u[k + n_b - 1] + ... + b_{n_b}u[k],$$  \hspace{1cm} (4.150)

where the order of the (4.150) is defined to be equal to the poles, that is \(n_a\). The zeros is defined to be \(n_b\). We also assume that \(n_a \geq n_b\). The pole excess is defined to be \(d = n_a - n_b\). Applying forward-shift operator on (4.150) gives

$$\left( z^{n_a} + a_1 z^{n_a-1} + a_2 z^{n_a-2} + ... + a_{n_a} \right) y[k] = (b_0 z^{n_b} + b_1 z^{n_b-1} + b_2 z^{n_b-2} + ... + b_{n_b}) u[k].$$  \hspace{1cm} (4.151)

Let us define the polynomials

$$A(z) = z^{n_a} + a_1 z^{n_a-1} + a_2 z^{n_a-2} + ... + a_{n_a},$$  \hspace{1cm} (4.152)

and

$$B(z) = b_0 z^{n_b} + b_1 z^{n_b-1} + b_2 z^{n_b-2} + ... + b_{n_b}.$$  \hspace{1cm} (4.153)

Hence, (4.150) can be reformulated as

$$A(z)y[k] = B(z)u[k].$$  \hspace{1cm} (4.154)

It is also possible to reformulate 4.150 such that

$$y[k] + a_1 y[k - 1] + ... + a_{n_a} y[k - n_a] = b_0 u[k + n_b - n_a] + b_1 u[k + n_b - n_a - 1] + ... + b_{n_b} u[k - n_a].$$ \hspace{1cm} (4.155)

By reversing the order of the coefficients of \(A(z)\), the reciprocal polynomial \(A^\ast(z)\) is defined

$$A^\ast(z) = 1 + a_1 z^2 + a_2 z^2 + ... + a_{n_a} z^{n_a},$$ \hspace{1cm} (4.156)

$$= z^n z^{-(n_a - 1)} + a_2 z^{-(n_a - 2)} + ... + a_{n_a},$$ \hspace{1cm} (4.157)

$$= z^{-n_a} A(z).$$ \hspace{1cm} (4.158)

Notice that

$$A^\ast(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + ... + a_{n_a} z^{-n_a},$$ \hspace{1cm} (4.159)

$$= z^{-n_a} A(z).$$ \hspace{1cm} (4.160)

For \(B(z)\) we have similarly

$$B(z) = z^{n_b} B^\ast(z^{-1}).$$ \hspace{1cm} (4.161)

Using reciprocal polynomials (4.154) can be written

$$A(z)y[k] = B(z)u[k],$$

$$\Downarrow$$

$$z^{n_a} A^\ast(z^{-1}) y[k] = z^{n_b} B^\ast(z^{-1}) u[k],$$

$$\Downarrow$$

$$A^\ast(z^{-1}) y[k] = z^{n_b - n_a} B^\ast(z^{-1}) u[k],$$

$$\Downarrow$$

$$A^\ast(z^{-1}) y[k] = B^\ast(z^{-1}) u[k + n_b - n_a].$$
Notice that $A^{**}(z)$ is not necessarily the same as $A(z)$. The polynomial $A(z)$ is said to be self-reciprocal if

$$A^*(z) = A(z). \quad (4.162)$$

**Example 4.14** Let $A(z) = z$ Then the reciprocal is $A^*(z) = z \cdot z^{-1} = 1$. The reciprocal of $A^*(z)$ is $A^{**}(z) = 1$, which of course is different from $A(z)$.

### 4.12 Relation Between Shift-Operator Calculus and $z$-Transform

Calculation with $z$-transform and shift-operator calculus are related to each other. However, it is important to be aware of that in shift-operator calculus $z$ denotes an operator that acts on a double sequences, while in $z$-transform $z$ denotes a complex variable. Due to this several authors make a distinction between by using $q$ instead of $z$ in shift-operator calculus. Consider the following example from Åström and Wittenmark [333] about pole-zero cancellations:

**Example 4.15** Consider the difference equation

$$y[k + 1] + ay[k] = u[k + 1] + au[k], \quad (4.163)$$

where $a$ can be an arbitrary real number. The difference equation of (4.163) has the solution

$$y[k] = (-a)^k y[0] + u[k]. \quad (4.164)$$

The pulse-transfer function, see (4.99), of (4.163) is

$$H(z) = \frac{z + a}{z + a} = 1. \quad (4.165)$$

The last equality is obtained because $z$ is a complex variable. We may misled to believe that (4.163) is identical to

$$y[k] = u[k]. \quad (4.166)$$

This is only true if the initial condition $y[0] = 0$. If shift-operator calculus is used in solving (4.163), we achieve

$$(z + a)y[k] = (z + a)u[k]. \quad (4.167)$$

Since $z$ here is an operator we can not formally divide by $(z + a)$.

**Remark 4.17** Hence, in $z$-transform calculus it is allowed to divide with an arbitrary expression. However, this is not allowed in shift-operator calculus unless special assumptions are made i.e. initial condition is equal to zero.
Chapter 5

Signal Quality Checking and Fault Detection

In this chapter the most important signal processing properties for the purpose of control will be treated. Accurate control performance depends on reliable sensor signals. Unreliable signals from the sensors will influence the control systems capability for accurate and safe performance. Therefore poor signal quality should be detected, and the available information should be utilized in an optimal way, see Figure 5.1. Each signal should be checked for errors and bad signals should be rejected in a signal processing module. For many safety critical marine operations redundancy in instrumentation is required by the authorities. This means that two or more sensors measuring the same state are installed and interfaced to the control system. The redundant sensors could be of same type, or they could be different. For instance in dynamic positioning having redundancy in measuring the vessel position in North-East coordinates, two Differential Global Positioning Systems (DGPS) may be installed. The GPS is a satellite based positioning system. Even better, one DGPS and one hydroacoustic positioning reference (HPR) system could be installed. The HPR is an underwater based positioning system. In the last case redundancy is also achieved both in hardware, software and in the measuring principle. As we will see in Section 2.9 the authorities requires three or more number of sensors for conducting dynamic positioning in the most safety critical operations where loss of position could cause fatal accidents, severe pollution or damages with major economic consequences. This is defined as equipment or consequence class 3 operation. This means that if one of the three sensors fails, the dynamic positioning operation must be stopped. In order to increase the overall availability and reduce the days of off-hire, the ship owner then may decide to install a fourth and even a fifth system. Having redundant sensor signals available put additional requirements to the signal processing concerning signal weighting (which one should be trusted most), and signal drifting (signal voting may be used to reject a sensor with large deviation). Hence, handling of multiple signals in a consistent manner is crucial ensuring a safe and optimal operation.

5.1 Testing of Individual Signals

A signal processing module (Figure 5.1) should perform quality checking of all sensor signals, position reference system signals and thrust device feedback signals. Single signal quality checking will include different tests such as (Grøvlen et al. [101]):
• Signal range testing.
• Variance testing.
• Wild point testing.

The set of the tests above may be performed for every signal. If a failure situation is detected, the signal is unusable and the signal should be rejected.

5.1.1 Windowing

Different formulas based on different techniques for windowing for calculating a signal variance can be used (Oppenheim et al. [215]). Windowing is the operation of taking a signal $x[k]$ and multiplying it by a finite-duration window signal $w[k]$. That is,

$$p[k] = x[k]w[k].$$  \hspace{1cm} (5.1)

Then $p[n]$ is also of finite duration. In practice we are only able to measure a signal $x[k]$ over a finite time interval, time window. The actual signal available for analysis is then

$$p[k] = \begin{cases} x[k], & -M \leq k \leq M \\ 0, & \text{otherwise}, \end{cases}$$  \hspace{1cm} (5.2)

where $-M \leq k \leq M$ is the time window.

Using a rectangular window the finite-duration window signal is written

$$w[k] = \begin{cases} 1, & -M \leq k \leq M \\ 0, & \text{otherwise}. \end{cases}$$  \hspace{1cm} (5.3)
One of the problems using rectangular window is that it may introduce ripples on the Fourier transform. This is related to the so-called Gibbs phenomenon. Other techniques using exponential window and Hanning window may improve this situation. For details see Oppenheim *et al.* [215] and *Electronics Engineer’s Handbook* [61].

5.1.2 Signal Range Testing

Most of the signals available have a defined range. An example of this is the gyro sensor, whose heading output is in the range of $0 - 360^\circ$. If the signal processing module receives a gyro signal outside this range, it assumes that the sensor signal is faulty and will therefore reject it. Thus it is required that the signal is within a set of allowed values defined by a minimum value $x_{\text{min}}$ and a maximum value $x_{\text{max}}$ according to

$$x[k] \in [x_{\text{min}}, x_{\text{max}}].$$

(5.4)

5.1.3 Variance Testing

The signal variance gives an indication about the variations in amplitude and frequency of a signal. For instance, a high level of measurement noise gives high variance and vice versa. However, the signal variance may also be high due to a high level of process noise, i.e. heavy seas. This is not a failure situation and should therefore be reflected in the calculations.

Consider the sequence $\{x[k]\}$ at $t = k$ consisting of the signal itself and $n - 1$ historical values according to

$$\{x[i] : i = k - (n - 1), ..., k - 1, k\}.\quad (5.5)$$

The average value of the sequence $\bar{x}_k$ can be calculated to be

$$\bar{x}_k = \frac{1}{n} \sum_{i=k-(n-1)}^{k} x[i].\quad (5.6)$$

The corresponding variance is then found to be

$$\sigma^2_k = \frac{1}{n-1} \left( \sum_{i=k-(n-1)}^{k} x[i]^2 - n\bar{x}_k^2 \right).\quad (5.7)$$

Let us define

$$y[k] = \frac{1}{n} \sum_{i=k+1-(n-1)}^{k} x[i]^2.\quad (5.8)$$

On recursive form (5.7) can be written as

$$\sigma^2_{k+1} = \frac{n}{n-1} \left( y[k+1] - (\bar{x}[k+1])^2 \right),\quad (5.9)$$

where

$$y[k+1] = y[k] + \frac{1}{n} \left( (x[k+1])^2 - (x[k - (n - 1)])^2 \right).\quad (5.10)$$

High variance may be a symptom of a sensor failure or inaccurate measurement. A frozen signal can indicate a failure in a sensor, leading to zero variance. Hence, both an upper and a lower limit is considered in the variance test, see Figure 5.2.
5.1.4 Wild Point Testing

If a wild point is detected, this measurement should be rejected for one sample. A wild point is a measurement that deviates considerably from the previous measurements, that is, it is outside a certain band about the estimated mean signal, see Figure 5.3. The sampled value $x[k]$ may be accepted if

$$x[k] \in [\bar{x}_k - a\sigma, \bar{x}_k + a\sigma],$$  \hspace{1cm} (5.11)

where $a$ often is set to be in the interval $3 - 9$. Normally, the wild point is replaced by a calculated value. This could be the value from the previous sample, the mean value or an estimated minimum variance value.

5.2 Handling of Redundant Measurements

For redundant sensor or position reference system configurations, the possibilities for fault detection, weighting and voting are improved. Signals from different sensors or position reference systems are compared and the difference between them will indicate a failure situation. Weighting principles are used for calculation of optimal input data to the controller algorithm.

5.2.1 Voting

The signal processing module provides two levels of voting for detection of drifting of sensors and position reference systems.

If two sensors or position reference systems are available, the signal processing module can detect drifting (bias) between the two sources. However, it is not able to detect which sensor or position reference system that has failed unless an individual signal erroneous situation is detected. Anyway, the operator receives a warning that a signal is detected.

If three or more sensors are available, the signal processing module is also able to carry out voting in order to detect whether one of the signals are truly drifting. Figure 5.4 illustrates this situation. The three measurements are different, but it seems that sensor no 2 deviates from sensor no 1 and no 3. The voting algorithm will reject sensor no 2 when the deviation exceeds a defined limit.

Figure 5.2: Signal variance test.
Figure 5.3: Wild point test.

Figure 5.4: Voting of redundant measurements.
5.2.2 Weighting

For redundant sensor or position reference system configurations the signal processing module performs a weighting of the individual signal before in order to calculate the optimal average signals. The individual signal weighting factors are either calculated automatically based on the signal variance or set manually. A sensor which has high variance compared to the other sensors will, in the automatic case, have a correspondingly low weighting factor and influence on the weighted signal. However, if the variance is too low (frozen signal) the signal is discarded from the weighting calculation.

The equation below illustrates calculation of a weighted sensor measurement, \( x_w \). Assume that three sensor measurements are available, \( x_1, x_2 \) and \( x_3 \), with weighting factors \( w_1, w_2 \) and \( w_3 \), then the weighted measurement becomes

\[
x_w = \frac{w_1 x_1 + w_2 x_2 + w_3 x_3}{w_1 + w_2 + w_3}.
\]  
(5.12)

Consider an unbiased estimated signal consisting of a sum of \( n \) independent weighted signal according to

\[
\hat{x} = \sum_{i=1}^{n} s_i x_i,
\]  
(5.13)

where

\[
\sum_{i=1}^{n} s_i = 1.
\]  
(5.14)

The weighting factors can be set manually by an operator or automatically calculated based on the principle of minimum variance. For manual weighting we have

\[
s_i = \frac{w_i}{\sum_{k=1}^{n} w_k}.
\]  
(5.15)

For automatic weighting consisting of two measurements the weights become

\[
s_1 = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2},
\]  
(5.16)

\[
s_2 = \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}.
\]  
(5.17)

Notice that a higher variance in one of the signals results in higher weight in the remaining signal. The general automatic weighting algorithm for \( n \) signals can be written

\[
s_i = \frac{\prod_{j \neq i} \sigma_j^2}{\sum_{k=1}^{n} \prod_{j \neq k} \sigma_j^2}.
\]  
(5.18)

5.2.3 Enabling and Disabling of Sensors

Operator initiated disabling or an abrupt loss of a sensor can lead to the signal deviation effect shown in Figure 5.5. This can be avoided by filtering the signal average a specific time period after the loss of signal has occurred, illustrated by Figure 5.6. The filter should be activated
subject to an event indicating the loss of the sensor signal. An appropriate time period $T_f$ depending on a signal difference and a maximum change rate should be specified. A change in the average value is inevitable after a signal loss but the filtering will prevent the unwanted response.

Let $y_w$ denote the new calculated weighted signal after the failure situation. Then, the filtered weighted signal $y_{fw}$ entering the control system could be found by solving the lowpass equation below

$$\dot{y}_{fw} = -\frac{1}{T_f} y_{fw} + \frac{1}{T_f} y_w.$$  \hspace{1cm} (5.19)

The filtering should not inflict a phase change of the measurement too much. When enabling a sensor, the signal will remain smooth and no filtering is necessary.
Chapter 6

Filtering and State Estimation

In order to be successful in the design of industrial control system in addition to appropriate signal processing, filtering of signals are essential. Topics considered in this chapter are basic introduction to:

- **Filtering of measurement noise.** Most sensor signals are contaminated by some noise caused by external disturbances and internal properties of the sensor itself. The noise may have negative impact on the controller performance, if no precaution is taken. By filtering we achieve a change in the relative amplitudes of the frequency components in a signal or even elimination of some frequency components entirely.

- **Reconstruction of non-measured data.** For many applications important process states are not measured. Typical reasons for this could be that no convenient sensors exists, or simply that cost reasons motivate to not to install the sensor. In such cases sophisticated model based filtering techniques - state estimation - can be applied. The main purpose of the state estimator (observer) is to reconstruct unmeasured signals and perform filtering before the signals are used in a feedback control system. The input to the state estimator in a dynamic positioning system is sensor data e.g. from an inertial measurement unit (IMU) measuring the vessel’s heave, roll and pitch motions, a compass or gyro measuring the vessel’s heading and a position reference system like the satellite navigation system DGPS (differential global positioning system) measuring the vessel’s North and East position coordinates, see Chapter 2 for more description of these sensors.

- **Dead reckoning.** All kind of equipment will fail according to some failure rate. Experience from industrial applications has shown that the most frequent control system failure are caused by sensor failures. In safety critical marine applications a sudden drop-out of the control system may lead to dangerous situations, if not an adequate signal substitution will take place. Applying model based filters the predicted sensor signal may, at least for some period of time, replace the measured signal. Then only the mathematical model of the process and the sensor device itself is used to predict the signal to be used by the control system. Using only model prediction for calculation of state signal is called dead reckoning. Use of observers for the purpose of diagnostics and in the design of so-called virtual sensors for fault-tolerant control is an increasing field of research.

This chapter will present the basic properties of the most used conventional analog and digital filters, and model based filters (also called observers) used in control system design. Most of the
theory presented is general and valid for almost all kind of industrial control problems. However, certain filtering and state estimation problems a more characteristics for marine applications. These will be paid more attention to later in the text in Chapter 8. The chapter is organized as follows: Section 6.1 will handle conventional analog and digital filtering techniques. In Section 6.2 model based state space filtering techniques also called observers will be presented.

The theory presented in this chapter is solely based on earlier text books. For more thoroughly presentation the reader is referred to Johansson [138], Egeland [63], Fossen [78], Electronics Engineer’s Handbook [61], Oppenheim et al. [215] and Åström and Wittenmark [333]. Concerning diagnostics and fault-tolerant control the reader is referred to Blanke et al. [31].

6.1 Analog and Digital Filtering

According to Oppenheim et al. [215] Linear time-invariant (LTI) systems that change the shape of the systems power spectrum are referred to as frequency-shaping filters. Filters that are designed to pass some frequencies essentially undistorted and significantly attenuate or eliminate others are called frequency-selective filters. Due to the broad field of applications we will below first concentrate on frequency-selective filters. Several basic types of filters has been designed, and they are given name indicating their main function. The most common ideal filters are:

- **Lowpass filter** passes low frequencies of signals from zero to cut-off frequency and attenuate or reject all higher frequencies.
- **Highpass filter** passes the high frequencies of signals and attenuate or reject all low frequencies from zero to cut-off frequency.
- **Bandpass filter** passes a band of frequencies between lower and upper cut-off frequencies and attenuate frequencies higher and lower than those defined by the band.
- **Bandstop filter** (or reject filter) stops signal frequencies between its lower and upper cut-off frequencies, and transmits all other signals.
- **All-pass filter** transmits all signal frequencies and produces a predictable phase shift.

**Example 6.1** A continuous-time ideal lowpass filter is a LTI system that passes complex exponential $e^{j\omega t}$ for values of $\omega$ in the range of $-\omega_c \leq \omega \leq \omega_c$ and rejects all other frequencies, where $\omega_c$ is the cut-off frequency. Hence, the frequency response is given on the form:

$$h(j\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & |\omega| > \omega_c \end{cases}.$$  \hspace{1cm} (6.1)

Adopting the complex exponential signal $e^{j\omega t}$, ideal filters are usually defined to have symmetrical frequency response about $\omega = 0$, where $|h(j\omega)| = 1$.

Ideal filters is useful to describe idealized system configurations. However, they are not realizable and must be approximated in real applications as nonideal filters. Nonideal filters do have a gradual transition band from passband to stopband. The acceptable deviation limits in the passband and the stopband are often described by $\delta_1$ and $\delta_2$, see Figure 6.1. The amount by which the frequency response differs from unity in the passband is referred to as passband ripple,
and corresponding deviation from zero in the stopband is referred to as stopband ripple. In addition, especially for control purposes the phase characteristics of the filter is of importance. For nonideal lowpass filters, there is a trade-off between the width of the transition band (frequency domain) and the rise time, delay time and overshoot (time domain) of the step response.

### 6.1.1 Nonideal Lowpass Filter

A widely-used class of LTI systems for lowpass filtering (Oppenheim et al. [215]) is the so-called $n^{th}$-order Butterworth filters. In the frequency domain for $n = 1, 2$ and $3$

\[
\begin{align*}
  n &= 1 : b(s) = \frac{\omega_c}{s + \omega_c}, \quad (6.2) \\
  n &= 2 : b(s) = \frac{\omega_c^2}{(s^2 + \sqrt{2}\omega_c s + \omega_c^2)}, \quad (6.3) \\
  n &= 3 : b(s) = \frac{\omega_c^3}{(s^2 + \omega_c s + \omega_c^2)(s + \omega_c)}, \quad (6.4)
\end{align*}
\]

The corresponding filters may also be written in time domain according to

\[
\begin{align*}
  n &= 1 : \ddot{x}_f + \omega_c x_f = \omega_c x, \quad (6.5) \\
  n &= 2 : \dddot{x}_f + \sqrt{2}\omega_c \ddot{x}_f + \omega_c^2 x_f = \omega_c^2 x, \quad (6.6) \\
  n &= 3 : \ddot{x}_f + 2\omega_c \dot{x}_f + 2\omega_c^2 x_f + \omega_c^3 x = \omega_c^3 x, \quad (6.7)
\end{align*}
\]

where $x$ is the signal to be filtered (input), $x_f$ is the filtered signal (output) and $\omega_c$ is the cut-off frequency. The Butterworth filters have a frequency response which is optimized to give as flat amplitude as possible in the passband, see Figure 6.2. Similarly other classes of lowpass filter exist, where the most known are Chebyshev filter, Bessel filter, Legendre-Papoulis filter and Elliptic filter, see Part 16 in *Electronics Engineer’s Handbook* [61] for more details. The Elliptic filter does also have zeros that may differ from infinite.
6.1.2 Nonideal Highpass Filter

A highpass filter may be designed by substituting

\[ s \rightarrow \frac{1}{s} \]

in the equations describing the lowpass filter.

6.1.3 Notch Filter

Cascaded notch filter is an often used bandstop filter and is written

\[ h_n = \prod_{i=1}^{r} \frac{s^2 + 2\zeta_{ni}\omega_i s + \omega_i^2}{s^2 + 2\zeta_{di}\omega_i s + \omega_i^2} \]

where \( \omega_i \) is the center frequencies of the filter, \( r \) is the number of center frequencies, and \( \zeta_{ni} \ll 1 \) and \( 0 < \zeta_{di} \) are the damping ratios in the numerator and the denominator, respectively. A typical choice could be \( \zeta_{ni} = 0.1 \) and \( \zeta_{di} = 1 \). Notch filter in cascade with lowpass filter have often in the past been used for wave filtering. An appropriate number of center frequencies, typically 3, with a distribution about the domination wave frequency is chosen, see Figure 6.3.

6.1.4 Digital Filtering

Digital filters provide many of the same frequency selective services as analog filters. Digital filters are often defined in terms of equivalent analog filters. However, many digital filters are
designed using properties that are unique to this technology. Compared to analog filters, digital filters generally have the advantages that the stability of certain classes can be guaranteed, coefficients can easily be altered, they can operate over a wide range of frequencies with wide dynamic range and with high precision.

Consider the equation

$$\dot{x} = u.$$  \hfill (6.10)

Approximation of this equation could be

$$\frac{x[k+1] - x[k]}{T} = (u[k+1] + u[k])\frac{1}{2}. \hfill (6.11)$$

Applying $z$-transform gives

$$\frac{(z-1)x[z]}{T} = \frac{1}{2}(z+1)u[z],$$

$$\Downarrow$$

$$x[z] = \frac{T(z+1)}{2(z-1)}u[z]. \hfill (6.12)$$

Hence, once an analog filter $h(s)$ is defined, the discrete filter $H(z)$ based on trapezoidal approximation method could be constructed by replacing $s$ in $h(s)$ by

$$s = \frac{2(z-1)}{T(z+1)}. \hfill (6.13)$$
Setting $z = e^{j\omega T}$ in (6.13) gives
$$s = \frac{2(e^{j\omega T} - 1)}{T(e^{j\omega T} + 1)} = \frac{2}{T} \tan \left( \frac{\omega T}{2} \right) = iv,$$
(6.14)
where $v$ is the discrete frequency corresponding to $\omega$. When $\frac{\omega T}{2} \to \frac{\pi}{2}$, $v \to \infty$. As noticed in (4.78) $\omega = \frac{\pi}{2}$ is known as the Nyquist frequency, which is the absolute upper limit for the discrete filter to approximate the analog filter.

Linear constant coefficients filters can be categorized into two main classes known as finite impulse-response (FIR) filter or infinite impulse-response (IIR) filters.

**Finite Impulse-Response (FIR) filter**

FIR filters can be expressed as
$$y[k] = b_0 u[k] + b_1 u[k-1] + \ldots + b_n u[k-n],$$
(6.15)
where the coefficients $\{b_i\}$ are called filter tap weights. In terms of $z$-transform the filter discrete transfer function is written
$$H(z) = \sum_{i=0}^{n} b_i z^{-i}.$$  
(6.16)

The filter’s transfer function consists of zeroes only (i.e. no poles). Because of this the FIR filter is referred to as an all-zero or transversal filter. If the filter input is bounded (i.e. $|u(i)| \leq 1$ for all $i$), the maximum value of the output $y(i)$ is $\sum |b_i|$. If all the tap weights $\{b_i\}$ are bounded, the filters output is likewise bounded and, as result, stability is guaranteed. Furthermore, the phase of $H(z)$, when plotted with respect to frequency, is linear with constant slope.

**Example 6.2** For moving average filter it is convenient to apply a FIR formulation, where the output $y[k]$ for any $k$, i.e. $k_0$, is an average of values $u[k]$ in the vicinity of $k_0$. An example of a two-point average filter is
$$y[k] = \frac{1}{2} (u[k-1] + u[k]).$$
(6.17)

Three-point moving average filter may be
$$y[k] = \frac{1}{3} u[k-1] + u[k] + u[k+1].$$
(6.18)

**Infinite Impulse-Response (IIR) filter**

IIR filters can be expressed as
$$y[k] = -a_1 y[k-1] - a_2 y[k-2] - \ldots - a_n y[k-n] + b_0 u[k] + b_1 u[k-1] + \ldots + b_n u[k-n],$$
(6.19)
where the the coefficients $\{a_i\}$ and $\{b_i\}$ are the filter tap weights. IIR filter generally satisfies a given magnitude frequency response design objective with a lower order filter compared to a corresponding FIR filter. However the phase characteristics does not generally exhibit linear phase. FIR filter can only be realized in digital form, while IIR filter may be realized analog.

The most known IIR filters are Tsjebyscheff, Elliptic and Butterworth. The discrete IIR version of a Butterworth filter may be derived putting (6.13) into (6.2)-(6.4).
6.2 State Estimation

Analog filters are not always suitable for all control applications. In many cases the signals to be separated do not appear in totally disjoint frequency bands. A disadvantage concerning control is that undesired phase lag often is introduced by filters. This consequently reduce the control performance and stability margins in terms of bandwidth reduction, and by less gain and phase margin. Thus for the purpose of control especially higher order lowpass filter or cascades of filters should be used with care. Very often the signals of interest consists of frequency components in different frequency band slightly overlapping each others. This is in particular the case for many marine applications.

In industrial control systems many of the states of interest concerning control are impossible to measure, or if possible, the cost for installing the sensors may be too high. It is therefore of importance to determine or reconstruct those states from the available measurements. This is called state estimation and is provided by so-called observers.

**Definition 6.1** An observer or state estimator produces the state of a system from measurements of inputs and outputs.

In positioning of marine vessels, the main purpose of the observer (Figure 6.4) is to estimate velocities and current and wave drift forces from position measurements. Later in the text in Section 8.2 we will see that, in addition, 1st-order wave disturbances should be filtered out e.g. by using a notch filter suppressing wave-induced disturbances close to the peak frequency of the wave spectrum. The position and heading measurements are corrupted with colored noise caused by wind, waves and ocean currents. However, only the slowly-varying disturbances should be counteracted by the propulsion system whereas the oscillatory motion due to the waves (1st-order wave-induced disturbances) should not enter the feedback loop. This is done by using so-called wave filtering techniques, which separates the position and heading measurements into a low-frequency (LF) and a wave-frequency (WF) position and heading part, see Figure 7.20. Even though accurate measurements of the vessel velocities are available when using differential GPS systems or Doppler log, a state estimator must be designed in order to satisfy the classification rules for the positioning systems. In the case of temporarily loss of position and heading measurements, the observer must be able to operate in a dead reckoning mode implying that predicted observer velocity, position and heading is used for feedback. A temporarily loss of these measurements will not affect the positioning accuracy. When the necessary signals reappear, the estimated values will give a smooth transition back to the true position and heading.

Before we address the marine applications any further, basic introduction to the most used linear observers will be given in this chapter.

### 6.2.1 Deterministic Estimators

In the ideal situation when there are no uncertainties in the system with regards to modelling parameters and noise, the system is called deterministic. For this simplified case, a deterministic estimator may be constructed to illustrate the concept of state estimation. In the realistic case with modelling uncertainties and noise on measurements and control signals, stochastic theory may be applied in the observer design. The best known stochastic estimator is the Kalman filter, which will be treated subsequently.
Continuous-time deterministic estimator

Consider the following control plant model of the linear time-invariant (LTI) system of the form

\[
\dot{x} = Ax + Bu, \quad y = Dx. \tag{6.20}
\]

This system is completely deterministic, and it is assumed that the pair \([A, D^T]\) is observable (actually, detectability is sufficient). This means that it is possible to estimate the system states from its inputs and outputs. A first attempt at constructing an estimator might be to require that the estimator copies the control plant model exactly

\[
\dot{\hat{x}} = A\hat{x} + Bu, \quad \hat{y} = D\hat{x}. \tag{6.22}
\]

where \(\hat{x}\) is the estimated state vector. But if the initial conditions \(x(t_0)\) and \(\hat{x}(t_0)\) differ, the estimator will not reproduce the correct states. To correct this, an injection term using the difference between the measured output \(y\) and the estimated output \(\hat{y}\) is added to the estimator

\[
\dot{\hat{x}} = A\hat{x} + Bu + K_e(y - \hat{y}) = A\hat{x} + Bu + K_e D(x - \hat{x}). \tag{6.24}
\]

The state-feedback gain matrix \(K_e\) may then be used for tuning the estimator. The error dynamics is found by subtracting (6.24) from (6.20)

\[
\frac{d}{dt}\tilde{x} = \dot{x} - \dot{\hat{x}} = A(x - \hat{x}) - K_e D(x - \hat{x}) = (A - K_e D)(x - \hat{x}) = (A - K_e D)\tilde{x}. \tag{6.25}
\]

Hence, the error \(\tilde{x}\) will exponentially approach zero if the eigenvalues of \((A - K_e D)\) have negative real parts. Figure 6.5 shows a block diagram of this estimator.

If \(K_e\) is chosen only with respect to obtaining fast convergence of \(\hat{x}\) to \(x\), the poles will be moved far away into the left half-plane. This will increase the bandwidth of the estimator, which means that \(\hat{x}\) will be more sensitive to noise in \(u\) and \(y\). The choice of \(K_e\) is thus a trade-off between fast estimation and noise-rejection properties, where the best compromise is achieved by an optimal estimator (Anderson and Moore [7]).
Figure 6.5: Deterministic estimator illustrating the plant model concept.

Discrete-time deterministic estimator

Consider the control plant model of the discrete system

\[
\begin{align*}
x[k+1] &= \Phi x[k] + \Delta u[k], \\
y[k] &= Cx[k].
\end{align*}
\] (6.26)

The estimator can be found by copying the control plant model such that

\[
\begin{align*}
\hat{x}[k+1] &= \Phi \hat{x}[k] + \Delta u[k] + L(y[k] - \hat{y}[k]), \\
\hat{y}[k] &= C\hat{x}[k],
\end{align*}
\] (6.28)

(6.29)

where \(L\) is the state-feedback gain matrix, and \(\hat{x}\) is the estimate of \(x\). The error dynamics become (with \(\bar{x} = x - \hat{x}\))

\[
\bar{x}[k+1] = \Phi \bar{x}[k] - L(y[k] - \hat{y}[k]) = \Phi \bar{x}[k] - L(\bar{x}[k] - \hat{x}[k]) = (\Phi - LC)\bar{x}[k].
\] (6.30)

Now \(L\) must be chosen such that (6.30) is asymptotically stable, which means that the magnitude of the eigenvalues of \((\Phi - LC)\) must be within the unit circle. This is again only possible if the system is observable.

Certainty equivalence principle

If a dynamic system is defined by (6.26) and (6.27), a feedback control law may be defined by using a complete state information and the positive gain matrix \(G > 0\) such that

\[
\begin{align*}
u[k] &= -Gx[k], \\
x[k+1] &= \Phi x[k] - \Delta Gx[k] = (\Phi - \Delta G)x[k].
\end{align*}
\] (6.31)

(6.32)
If the complete state information is unavailable, the observer defined in (6.28) may be used, and the estimated state vector \( \hat{x} \) replaces \( x \) in the calculation of the control input according to \( u = -G\hat{x}[k] \). This is in the literature denoted as the certainty equivalence principle or separation theorem. The closed loop error dynamics of the complete feedback system becomes

\[
\begin{align*}
  x[k+1] - \hat{x}[k+1] &= (\Phi - \Delta G)x[k] - (\Phi - \Delta G)\hat{x}[k] - LC\hat{x}[k] \\
  &= (\Phi - LC - \Delta G)\hat{x}[k].
\end{align*}
\]

(6.33)

This system is of order \( 2n \) (with \( n \) as the order of the control plant model). The eigenvalues of the matrix \( \text{eig}(\Phi - \Delta G) \) is the eigenvalues of the closed-loop pole-placement problem with full state information. The eigenvalues of the matrix \( \text{eig}(\Phi - LC) \) of the observer is assumed to be faster than those of the controller, typical one decade. The solution of these two problems are dual, and the stability of the two systems may be analyzed separately (Åström and Wittenmark [333]). This principle of duality may be extended to more complex combinations of controllers and observers, and is a subject for research for nonlinear control and observer designs.

### 6.2.2 Least Squares Estimation

This section is partly based on Henriksen [112]. The state vector \( x \) of a static, linear stochastic process may be estimated from the measurement vector \( y \) and an a priori estimate of the state vector \( \bar{x} \), made prior to the time of the measurement. Let the measurement be described by

\[
y = Cx + v,
\]

(6.34)

where \( C \) is a known matrix, and \( v \) is the measurement noise, which is assumed statistically independent of \( x \) with a known, positive definite covariance matrix \( V \) and expectation value equal to 0 such that

\[
E[v] = 0, \quad \text{cov}[v] = E[vv^T] = V > 0, \quad E[xv^T] = 0.
\]

(6.35)

The uncertainty of the a priori estimate \( \bar{x} \), which also is the expectation value of the corrected state vector \( x \), is given by the covariance matrix \( \bar{X} \)

\[
E[x] = \bar{x}, \quad \text{cov}[x] = E[(x - \bar{x})(x - \bar{x})^T] = \bar{X} > 0.
\]

(6.36)

A weighted least squares method for finding \( x \) from \( \bar{x} \) and \( y \) is given by

\[
J = \frac{1}{2}[(x - \bar{x})^T P(x - \bar{x}) + (y - Cx)^T Q(y - Cx)],
\]

(6.37)

where \( P \) and \( Q \) are appropriately chosen positive semidefinite and positive definite weighting matrices, respectively. By requiring the derivative of \( J \) to be zero, the value \( \hat{x} \) of \( x \) which minimizes \( J \) may be found

\[
\frac{dJ}{dx} \bigg|_{(x=\hat{x})} = P(\hat{x} - \bar{x}) - C^T Q(y - C\hat{x}) = 0,
\]

(6.38)

\[
(P + C^T QC)\hat{x} = P\bar{x} + C^T Qy = (P + C^T QC)\bar{x} + C^T Q(y - C\bar{x}),
\]

(6.39)

\[
\hat{x} = \bar{x} + (P + C^T QC)^{-1}C^T Q(y - C\bar{x}) = \bar{x} + K(y - C\bar{x}).
\]

(6.40)
The covariance matrix of the process disturbance is positive definite. Processes with known covariance matrices and white noise where it is assumed that the system is observable. Let us assume that the process noise \( \xi_i \) is defined as

\[
\xi_i = x_i - \bar{x}_i
\]

The mean value of the process disturbance \( E[\xi] = E[xv^T] = 0 \). One (important) way of choosing \( P \) and \( Q \) is to minimize the sum of diagonal terms in the covariance matrix \( \hat{X} \). It can be shown that this is achieved by the choice (Henriksen [112])

\[
P = \hat{X}^{-1}, \quad Q = V^{-1}.
\]

The gain matrix \( K \) is then given by

\[
K = (\hat{X}^{-1} + C^T V^{-1} C)^{-1} C^T V^{-1} = \hat{X} C^T (C \hat{X} C^T + V)^{-1}.
\]

It may be shown that the estimator \( \hat{x} \) is unbiased, which means that the expectation value of the \textit{a-posteriori} estimate equals the \textit{a priori} expectation value

\[
E[\hat{x}] = E[x] = \bar{x}.
\]

These results may now be used to develop the discrete Kalman Filter.

### 6.2.3 Discrete Kalman Filter

Consider the following discrete LTI system on the form

\[
x[k + 1] = \Phi x[k] + \Delta u[k] + \Gamma w[k],
\]

\[
y[k] = C x[k] + v[k],
\]

where it is assumed that the system is observable.

Let us assume that the process noise \( w[k] \) and the sensor noise \( v[k] \) are discrete Gaussian \textit{white noise} processes with known covariance matrices.

That is, \( w[k] \sim N(\bar{w}, W[k]) \) and \( v[k] \sim N(\bar{v}, V[k]) \). The mean value of the process disturbance is defined as

\[
E[w[k]] = \bar{w}.
\]

The covariance matrix of the process disturbance is positive definite (Appendix A) and is defined as

\[
E[(w[k] - \bar{w})(w[k] - \bar{w})^T] = W[k] \delta_{kj},
\]
where $\delta_{kj} = 1$ if $k = j$ and $\delta_{kj} = 0$ if $k \neq j$. The mean value of the sensor noise is defined as
\[ E[v[k]] = \bar{v} = 0. \tag{6.48} \]
The covariance matrix of the sensor noise is positive definite and defined as
\[ E[(v[k] - \bar{v})(v[k] - \bar{v})^T] = V[k] \delta_{kj}. \tag{6.49} \]
Assume that the process disturbance and measurement noise are uncorrelated:
\[ E[v[k]w[k]^T] = 0. \tag{6.50} \]
The expectation value of the initial condition of the state is defined as
\[ E[x[0]] = \bar{x}(0). \tag{6.51} \]
The corresponding covariance matrix of the initial condition of the state is positive definite and is defined as
\[ E[(x[0] - \bar{x}(0))(x[0] - \bar{x}(0))^T] = \bar{X}_0. \tag{6.52} \]
Assume that the process disturbance and the sensor noise is uncorrelated with the initial condition according to
\[ E[(x[0] - \bar{x}(0))w[k]^T] = 0, \tag{6.53} \]
\[ E[(x[0] - \bar{x}(0))v[k]^T] = 0. \tag{6.54} \]

**Remark 6.1** Notice that if the real disturbance is not a Gaussian process, it can be reformulated in a disturbance model (approximation) which is driven by white noise. The disturbance model is then included in an augmented state vector.

Assume that the data
\[ Y_k = \{y[i], u[i] \mid i \leq k\}, \tag{6.55} \]
is known. Using $Y_k$ it is possible to have three cases for estimation (Figure 6.6) of $x[k + m]$:
- Smoothing for $m < 0$.
- Filtering for $m = 0$.
- Prediction for $m > 0$.

We will first consider the case when $m = 1$. This is a so-called one-step prediction. Then the filtering case with $m = 0$ will be shown.
Kalman filter - prediction case

Let us denote the vector $\hat{x}$ to be the state estimate of the state vector $x$. Consider the system given by (6.44) and (6.45) and assume that the estimated state vector is a function of past inputs according to

$$\hat{x}[k+1|k] = \Phi \hat{x}[k|k-1] + \Delta u[k] + \Gamma \bar{w}[k] + K[k] (y[k] - \hat{y}[k]), \quad (6.56)$$

$$\hat{y}[k] = C \hat{x}[k|k-1]. \quad (6.57)$$

A new feedback term $K[k] (y[k] - \hat{y}[k])$ is introduced. $K[k]$ is a $n \times n$ gain matrix and is also called Kalman filter gain matrix if it is chosen subject to some principles as we will see later in the text. The notation $\hat{x}[k+1|k]$ is used to indicate that is is an estimate of $x[k+1]$ based on measurements available at time $k$. As long as the predicted measurement vector $\hat{y}[k]$ coincides exactly with the measurement vector $y[k]$ the feedback term will give no contribution. We often call the difference $y[k] - \hat{y}[k]$ to be the innovation. Eq. (6.56) and (6.57) is an observer of the LTI system (6.44) and (6.45).

The error dynamics of the reconstruction (estimation) can be found by subtracting (6.44) from (6.56) according to

$$\dot{\hat{x}}[k+1|k] = x[k+1] - \hat{x}[k+1|k], \quad (6.58)$$

$$\dot{\hat{x}}[k+1|k] = \Phi \dot{\hat{x}}[k|k-1] + \Gamma (w[k] - \bar{w}[k]) - K[k] (y[k] - \hat{y}[k]), \quad (6.59)$$

$$\dot{\hat{x}}[k+1|k] = (\Phi - K[k]C) \dot{\hat{x}}[k|k-1] + \Gamma (w[k] - \bar{w}[k]) - K[k]v[k], \quad (6.60)$$

$$\dot{\hat{x}}[k+1|k] = (I - K[k]) \left( \begin{array}{c} \Phi \\ C \end{array} \right) \dot{\hat{x}}[k|k-1] + \left( \begin{array}{c} \Gamma (w[k] - \bar{w}[k]) \\ v[k] \end{array} \right). \quad (6.61)$$

**Remark 6.2** If we disregard the effect of $\Gamma (w[k] - \bar{w}[k]) - K[k]v[k]$, we observe that the choice of $K[k]$ can be made such that (6.61) is asymptotically stable. Thus, the error dynamics converges...
to zero even if the system itself is unstable providing that the pair \((\Phi, C)\), the criterion for observability is fulfilled. Hence, the \(n \times n\) observability matrix \(Q_{do}\) has full rank, see (4.60) in Section 4.4.

However, we will here take the properties of the noise into account in the design of \(K[k]\) minimizing the variance of the error dynamics, which is denoted as

\[
\bar{X}[k] = E[(\bar{x}[k] - \bar{E}(\bar{x}[k]))(\bar{x}[k] - \bar{E}(\bar{x}[k]))^T].
\]

(6.62)

The expectation value of the error \(\bar{x}\) is

\[
E[\bar{x}[k + 1]] = (\Phi - K[k]C)E[\bar{x}[k]].
\]

(6.63)

Let \(E[x[0]] = \bar{x}(0)\). If \(\bar{x}[0] = \bar{x}(0)\), then \(E[\bar{x}[0]] = 0\). Thus the mean value of the error dynamics is zero for all \(k \geq 0\) independent of \(K[k]\). Since \(E[\bar{x}[k]]\) is assumed to be independent of \(w[k]\) and \(v[k]\), (6.61) gives

\[
\bar{X}[k + 1] = E[(\bar{x}[k + 1] - \bar{E}(\bar{x}[k + 1]))(\bar{x}[k + 1] - \bar{E}(\bar{x}[k + 1]))^T],
\]

(6.64)

Further \(\bar{X}[0] = \bar{X}_0\). From (6.64) it follows that if \(\bar{X}[k]\) is positive semidefinite, then \(\bar{X}[k + 1]\) is also positive semidefinite.

Let us first define a general principle called the completion of squares (Åström and Wittenmark [333]), where the quadratic loss function is defined as

\[
J(x - \hat{x}, y - \hat{y}) = (x - \hat{x})^T(Q_y^{-1}Q_{xy}(y - \hat{y}))^T(\begin{bmatrix} Q_x & Q_{xy} \\ Q_{xy}^T & Q_y \end{bmatrix})(x - \hat{x}, y - \hat{y}),
\]

(6.65)

where \(Q_y > 0\) is a symmetric positive definite matrix and \(Q_x \geq 0\) is a symmetric positive semidefinite matrix. The minimum of (6.65) with respect to \(x\) is found by

\[
\frac{\partial J(x - \hat{x}, y - \hat{y})}{\partial x} \bigg|_{x=\hat{x}} = 0.
\]

(6.66)

Then there exists an \(K\) satisfying

\[
Q_y KC = Q_{xy}^T,
\]

(6.67)

such that (6.65) can be rewritten

\[
J(x - \hat{x}, y - \hat{y}) = (x - \hat{x})^T(Q_x - (KC)^TQ_yKC)(x - \hat{x}) + (y - \hat{y} + KC(x - \hat{x}))^TQ_y(y - \hat{y} + KC(x - \hat{x})).
\]

(6.68)

(6.69)

Because (6.68) is quadratic in \(y - \hat{y}\) and both terms are greater or equal to zero, it is seen that (6.68) is minimized for

\[
y - \hat{y} = -KC(x - \hat{x}).
\]

(6.70)
Thus, it can be shown that $K$ is unique since $Q_y > 0$. The minimum is therefore

$$J(x - \hat{x}, y - \hat{y}) = (x - \hat{x})^T \left( Q_x - (KC)^T Q_y KC \right) (x - \hat{x}). \quad (6.71)$$

By using the idea of completion of squares, it follows that $\alpha^T \tilde{X}[k+1] \alpha$ is minimized by $K[k]$ satisfying

$$K[k] \left( V[k] + C\tilde{X}[k]C^T \right) = \Phi \tilde{X}[k]C^T, \quad (6.72)$$

for any $\alpha$. If $V[k] + C\tilde{X}[k]C^T$ is positive definite then

$$K[k] = \Phi \tilde{X}[k]C^T \left( V[k] + C\tilde{X}[k]C^T \right)^{-1}. \quad (6.73)$$

This inserted into (6.64) gives

$$\tilde{X}[k+1] = \Phi \tilde{X}[k] \Phi^T + \Gamma W[k] \Gamma^T - \Phi \tilde{X}[k]C^T \left( V[k] + C\tilde{X}[k]C^T \right)^{-1} C\tilde{X}[k] \Phi^T. \quad (6.74)$$

**Theorem 6.1** Åström and Wittenmark [333]. The Kalman filter predictor case is defined by (6.56) and (6.57). It is optimal with respect to minimum variance of the error dynamics if $(V[k] + C\tilde{X}[k]C^T)$ is positive definite, the disturbance and measurement error are Gaussian, and if the Kalman filter gain matrix is chosen as (6.73), where the covariance matrix of the error dynamics $\tilde{X}[k+1]$ is found by (6.74).

**Remark 6.3** As long as $E[v[k]] = 0$ it is possible to show that the an unbiased estimate is possible to achieve.

**Remark 6.4** Another notation of (6.74) is $\tilde{X}[k|k-1]$, indicating that the measurements up to and including $k - 1$ are used. The term $\Phi \tilde{X}[k-1] \Phi^T$ shows the influence of system dynamics on the covariance matrix of the error dynamics. $\Gamma W[k] \Gamma^T$ shows that the covariance matrix will increase for increasing process disturbance. The term

$$-\Phi \tilde{X}[k-1]C^T \left( V[k-1] + C\tilde{X}[k-1]C^T \right)^{-1} C\tilde{X}[k-1] \Phi^T, \quad (6.75)$$

shows how the variance is decreased accounting for information obtained through the measurements.

**Remark 6.5** Since $\tilde{X}[k|k-1]$ does not depend on any measurements and all other terms are assumed to be known, it is possible to precompute (6.73) and $\tilde{X}[k|k-1]$.

**Remark 6.6** For LTI where $\Phi$, $\Delta$, $\Gamma$ and $C$ are constant stationary solution of (6.73) can be found, where $\tilde{X}[k+1] \to \tilde{X}_\infty$ such that $K[k] \to K_\infty$. 

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Kalman filter - filter case

Consider the LTI process given by (6.56) and (6.57). If the matrix $\begin{pmatrix} V[k] + C\bar{X}[k-1]C^T \end{pmatrix}$ is positive definite, then the optimal filter can be found. The so-called corrector or aposteriori estimate is given by

$$\hat{x}[k|k] = \bar{x}[k|k-1] + K_f[k] (y[k] - C \bar{x}[k|k-1]).$$  \hspace{1cm} (6.65)

The predictor or apriori estimate is given by

$$\bar{x}[k+1|k] = \Phi \bar{x}[k|k] + \Delta u[k] + \Gamma \bar{w}[k|k],$$
$$= \Phi \bar{x}[k|k-1] + \Delta u[k] + \Gamma \bar{w}[k|k] + K[k] (y[k] - C \bar{x}[k|k-1]),$$  \hspace{1cm} (6.67)

where

$$K_f[k] = \bar{X}[k|k-1]C^T \left( V[k] + C\bar{X}[k|k-1]C^T \right)^{-1},$$
$$K[k] = \Phi K_f[k].$$ \hspace{1cm} (6.69)

The covariance matrix of the estimation error dynamics is given by a so-called Ricatti equation according to

$$\bar{X}[k+1|k] = \Phi \bar{X}[k|k-1] \Phi^T + \Gamma W[k] \Gamma^T - K[k] \left( V[k] + C\bar{X}[k|k-1]C^T \right) K[k] \right)^T \hspace{1cm} (6.68)$$
$$\bar{X}[k|k] = \bar{X}[k|k-1] - \bar{X}[k|k-1]C^T \left( V[k] + C\bar{X}[k|k-1]C^T \right)^{-1} C\bar{X}[k|k-1] \right)^T \hspace{1cm} (6.69)$$
$$\bar{X}[0|1] = \bar{X}_0.$$ \hspace{1cm} (6.70)

In Figure 6.7 the Kalman filter is illustrated graphically.

**Remark 6.7** The notation $\bar{X}[k|k-1]$ is used to specify the available data. Notice that $\bar{X}[k|k]$ is the variance of the estimation error dynamics at time $k$ given $Y_k$.

**Remark 6.8** The innovation given by $\xi[k] = y[k] - \hat{y}[k] = y[k] - C \bar{x}[k|k-1]$ will be white noise in a Kalman filter given as above.

**Remark 6.9** In dead reckoning (with a failure in sensor $i$ and thus loss of $y_i[k]$) the corresponding innovation will be set to $\xi_i[k] = 0$. Hence, only the predictor (6.77) will be active. Usually the predicted state $\bar{x}_i[k]$ becomes less noisy and sluggish without the corrector. Depending on the external disturbances and the accuracy of the model, the control performance may be satisfactory for quite long time. In dynamic positioning dead reckoning with satisfactory control performance up to several minutes has been experienced.

### 6.2.4 Extended Kalman Filtering

In the previous section we assumed that the process dynamics could be described by a LTI system. However, in many cases the process will be nonlinear. In marine applications conduction dynamic positioning the kinematics relating Earth-fixed coordinates to vessel-fixed coordinates represents such an nonlinearity. This will be explained more in detail later on. A common way to circumvent the problem of nonlinearities is to introduce a so-called extended Kalman filter,
which is briefly described in the following. Consider a nonlinear time-variant state-space model on discrete form

\[
\begin{align*}
x[k+1] &= f_k(x[k], u[k]) + \Gamma w[k] \\
y[k] &= h_k(x[k]) + v[k]
\end{align*}
\] (6.83)

In the extended Kalman filter, linearizations of the nonlinear dynamics \( f[k] \) and measurement \( h[k] \) are made about the current state estimate according to

\[
\begin{align*}
\Phi[k] &= \left. \frac{\partial f_k(x[k], u[k])}{\partial x[k]} \right|_{x[k]=\hat{x}[k]} \\
C[k] &= \left. \frac{\partial h_k(x[k])}{\partial x[k]} \right|_{x[k]=\hat{x}[k]} 
\end{align*}
\] (6.84) (6.85)

\( \Phi[k] \) and \( C[k] \) may then be substituted for \( \Phi \) and \( C \) in the equations for the discrete Kalman filter (see above) such that the Kalman filter gain matrix \( K \) must be updated for each linearization, see above. In practice, we often make local linearized models about a set of predefined operating points and then apply individual precalculated Kalman filters for each model. By using proper gain scheduling techniques the correct filter model will be applied. The reader is referred to Bagchi [13] for details.
Chapter 7

Mathematical Modeling of Dynamically Positioned Marine Vessels

Depending on the operational conditions the vessel models may briefly be classified into station keeping, low velocity and high velocity models (Figure 7.1). Considering vessel motions in waves is defined as sea keeping and will here apply both for station keeping (zero speed) and forward speed. Dynamically positioning (DP) of vessels or thruster assisted position mooring (PM) of anchored marine vessels can in general be regarded as station keeping and low velocity or low Froude number applications. This assumption will particularly be used in the formulation of mathematical models used in conjunction with the controller design. Notice that the manoeuvring models often assumes low-frequency motions only, where the effect of wave-frequency motions is included afterwards by linear superposition.

Easy access to computer capacity, and the presence of efficient control system design toolboxes, such as Matlab, Simulink and others, have motivated more extensive use of numerical simulations for design and verification of control systems. Essential in being successful in this is the ability to make sufficiently detailed mathematical models of the actual plant or process. From an industrial point of view, the same tendency is driven by the fact that control system safety and performance testing by hardware-in-the-loop (HIL) simulations contribute to reduced time for tuning during commissioning and sea trials and, not at least, reduced risk for incidents during operation caused by software bugs and erroneous control system configurations.

In controller design it is important to consider any coupling effects between the different degrees of freedom and subsystems, in addition to the natural periods (resonances) of the dynamic system, e.g. vessel, riser and mooring in open-loop (without control) and closed-loop (with control). The coupling effects and natural periods may be excited by the environmental loads or unintentionally by the control system itself if not properly accounted for in the design process. In Figure 7.2, typical natural periods for ships, semisubmersibles, risers and mooring system are given in addition to periods of wave loads. The working range or the bandwidth of a DP system is also indicated.

For the purpose of model-based observer and controller design, it is sufficient to derive a simplified mathematical model, which nevertheless is detailed enough to describe the main physical characteristics of the dynamic system. Then, structural information of the physical properties of the dynamic system is incorporated in the controller design in order to achieve a
Station keeping models
• Marine operation models
• Slender structures
• Multibody operations

Manoeuvring models
• Linearized about some $U_o$
• Sea keeping
• Motion damping

Figure 7.1: Modelling properties.

Figure 7.2: Periods of interest.
better performance and robustness compared to the conventional PID control design methods. For processes dominated by nonlinearities, it has also been experienced that nonlinear model-based control design will simplify the overall control algorithm since linearizations about different working points is avoided.

Hence, the mathematical models may be formulated in two complexity levels:

- **Control plant model** is a simplified mathematical description containing only the main physical properties of the process or plant. This model may constitute a part of the controller. The control plant model is also used in analytical stability analysis based on e.g. Lyapunov stability and passivity.

- **Process plant model** or **simulation model** is a comprehensive description of the actual process and should be as detailed as needed using high fidelity models. The main purpose of this model is to simulate the real plant dynamics. The process plant model is used in numerical performance and robustness analysis and testing of the control systems.

Due to the lack of process knowledge and thereby proper models, control plant models are often used as process plant models. This may result in bad controller designs, since "perfect" models are assumed. In Goodwin *et al.* [95] *calibration model* is used to describe the process plant model and *nominal model* is used to describe the control plant model.

This section will focus on the formulation of **process plant models**. In this context it means mathematical models of the vessel dynamics and external forces and moments, in terms of environmental loads, thruster/propeller forces and mooring forces (if any), acting on the vessel. Description of the **control plant models** will be presented in the controller design sections.

### 7.1 Environmental Models

Environmental models are essential for achieving realistic simulations of a marine vessel. Section 7.3 describes the environmental loads on the vessel, whereas this section describes environmental models for waves, wind and current.

#### 7.1.1 Waves

Irregular waves are commonly described by a wave spectrum \( S(\omega, \psi) \), which in general is a function of both frequency \( \omega \) and direction \( \psi \) (Myrhaug [194]). The wave spectrum is often divided in a frequency spectrum \( S(\omega) \) and a directional spreading function \( D(\psi, \omega) \), such that

\[
S(\omega, \psi) = S(\omega)D(\psi, \omega). \tag{7.1}
\]

The integral over all frequencies and directions represents the total energy in the sea state \( S_{tot} \)

\[
S_{tot} = \int_0^{2\pi} \int_0^{\infty} S(\omega, \psi)d\psi d\omega. \tag{7.2}
\]

Linearity is assumed, so that the harmonic wave components extracted from the spectrum may be superposed.
Figure 7.3: Wave spectrum as function of frequency.

**Frequency spectrum**

The frequency spectrum $S(\omega)$ describes the energy distribution of the sea state over different frequencies, with the integral over all frequencies representing the total energy of the sea state. From a given spectrum, a set of harmonic wave components representing the total energy may be extracted by choosing a set of frequencies $(\omega_q)$ and frequency intervals $(\Delta\omega_q)$ such that the whole area below the spectral curve is covered, see Figure 7.3. In the limiting case, where the number of wave components $n \to \infty$, and such $\Delta\omega_q \to 0$, this approaches the Riemann integral over $S(\omega)$. The amplitude $\zeta_{aq}$ of one wave is determined from the spectral value $S(\omega_q)$ by the formula

$$\zeta_{aq} = \sqrt{2S(\omega_q)\Delta\omega_q}. \quad (7.3)$$

In the following we will use a constant frequency interval $\Delta\omega$. The harmonic waves are assumed to have phase angles $(\phi_q)$ that are evenly distributed between 0 and $2\pi$. Each component is then described by

$$\zeta_q(t) = \zeta_{aq} \cos(\omega_q t + \phi_q). \quad (7.4)$$

Note that these harmonic waves represent the total wave energy with waves in only one direction. To include different wave directions, a spreading function must be added. The maximum frequency included in the realization of the wave spectra is denoted $\omega_{\text{max}}$. The frequency spectrum is commonly defined in terms of the significant wave height $H_s$ and the peak frequency $\omega_p$. $H_s$ is the average height of the 1/3 largest waves in the sea state, and $\omega_p$ is the frequency that contains the most energy in the sea state (i.e. defines the peak of the frequency spectrum). Notice that the wave height is defined as the distance from the bottom of a wave trough to the top of a wave peak, and hence is twice the wave amplitude.
Figure 7.4: Spreading function.

**Spreading function**

The spreading function $D(\psi, \omega)$ gives the directional distribution of wave energy, spanned about the mean wave direction $\psi_\omega$. It is in general a function of both direction and frequency, but is often simplified to be a function of only direction. The integral over all directions $(-\pi, \pi)$ must be unity, so that the total wave energy in the sea state is kept unchanged. To obtain "scaling factors" for the wave amplitudes in different directions, a set of directions $(\psi_i)$ and direction intervals $(\Delta \psi_i)$ must be chosen such that the whole area below the curve is covered, in the same way as with the frequency spectrum, see Figure 7.4. In the following we will use a constant direction interval $\Delta \psi$.

**Wave spectrum**

The wave spectrum $S(\omega, \psi)$, composed of the frequency spectrum and spreading function as in (7.1), now represents the energy distribution over both frequencies and directions. The integral over all directions equals the frequency spectrum $S(\omega)$, and the double integral over all directions and frequencies equals the total wave energy

$$\int_0^\infty \int_{\psi_0 - \pi}^{\psi_0 + \pi} S(\omega, \psi)d\psi d\omega = \int_0^\infty S(\omega)d\omega. \tag{7.5}$$

Figure 7.5 shows a wave spectrum, where each square composed of a $\Delta \omega$ and a $\Delta \psi$ represents one harmonic wave amplitude. The wave amplitude $\zeta_{aqr}$ is found by a modification of (7.3)

$$\zeta_{aqr} = \sqrt{2S(\omega_q, \psi_r)\Delta \omega \Delta \psi}. \tag{7.6}$$

One harmonic wave component is now represented by four parameters: Direction $\psi_r$, frequency $\omega_q$, amplitude $\zeta_{aqr}$ and phase angle $\phi_{qr}$. Note that the phase angles of all wave components are evenly distributed between 0 and $2\pi$. In an Earth-fixed coordinate system, the surface elevation in the coordinate $(x, y)$ will be given as

$$\zeta_{qr}(x, y, t) = \zeta_{aqr} \sin[\omega_q t + \phi_{qr} - k_y (x \cos \psi_r + y \sin \psi_r)]. \tag{7.7}$$

$k_y$ is the wave number, which equals $2\pi/\lambda_y$, where $\lambda_y$ is the wave length. For deep water, the dispersion relation $\omega_q^2 = k_y g$, where $g$ is the acceleration of gravity, gives the wave number from
Figure 7.5: Wave spectrum as function of frequency and direction.
the wave frequency. The coordinate system is defined so that waves with zero heading (ψᵣ = 0) are propagating towards north. The total surface elevation of all wave components at the point (x, y) at time t for N frequencies and M directions will then be

\[ \zeta(x, y, t) = \sum_{q=1}^{N} \sum_{r=1}^{M} \sqrt{2S(\omega_q, \psi_r)\Delta \omega \Delta \psi \sin(\omega_q t + \phi_{qr} - k_q(x \cos \psi_r + y \sin \psi_r))}. \]  

(7.8)

Observe that similar equations in e.g. Faltinsen [66] are given with different coordinate system conventions, and thus differ from the equation above.

**Implementation considerations**

Several modifications of the wave spectrum defined by (7.8) can be made in order to facilitate more realistic simulations when it is desired to keep the number of wave components as low as possible (and hence reduce computational effort).

**Random frequencies and directions** The expression in (7.8) will repeat itself after the time \( T = \frac{2\pi}{\Delta \omega} \). This means that a large number of wave components should be used, but as this is computationally demanding, a better solution is to choose a random frequency within each frequency interval \((\omega_q - \Delta \omega/2, \omega_q + \Delta \omega/2)\) (Faltinsen [66]). To further increase realism, the direction may also be chosen at random within each direction interval \((\psi_r - \Delta \psi/2, \psi_r + \Delta \psi/2)\).

The number of wave components is according to Faltinsen still recommended to be as high as 1000.

**Removing insignificant components** If implementing (7.8), with or without random frequencies and directions, the total number of harmonic wave components \( n_{\text{grid}} \) is the product of the number of directions in the grid and the number of frequencies in the grid, \( n_{\text{grid}} = N \cdot M \). However, both for high and low frequencies, as well as for directions far from the mean wave directions, the wave spectrum contains little energy. This means that the wave components generated from these areas have small amplitudes, and hence contribute little to the total sea state.

A way of reducing computational effort is therefore to avoid having too many wave components with close to zero amplitude. A first step at solving this is to choose the maximum frequency of the realization of the spectrum, \( \omega_{\text{max}} \), as low as possible, see Figure 7.3, and thereby avoid including insignificant wave components from the tail of the frequency spectrum. An easy way of implementing this is to do a test of the energy content of each wave component, and then discard the components with little energy. Two ways of doing this is:
1. Choose a fixed number of waves, \( n_{\text{waves}} \), which naturally must satisfy \( n_{\text{waves}} \leq n_{\text{grid}} \). Find the energy \( S(\omega_q, \psi_q) \) of each wave component and sort them in descending order. Then choose the \( n_{\text{waves}} \) first components for the simulation. The upside to this approach is that you get a user-defined number of waves. The downside is that you have no guarantee of not losing any significant wave components. If \( n_{\text{waves}} = n_{\text{grid}} \), all wave components are used.

2. Choose a relative wave component energy limit \( \kappa \in (0, 1) \) and discard all wave components which contain less than this ratio of the total sea state energy \( S_{\text{tot}} \):

\[
\frac{S(\omega_q, \psi_q)}{S_{\text{tot}}} < \kappa \Rightarrow \text{component discarded.} \tag{7.10}
\]

The upside to this approach is that you are guaranteed that all components with energy above the user-defined limit is kept. The downside is that you don’t have control over the number of waves. If \( \kappa = 0 \), all wave components are used.

### Common wave spectra

Common frequency spectra are the Pierson-Moskowitz (PM) spectrum, the ITTC/ISSC spectrum, the JONSWAP spectrum, and the more recent doubly peaked spectrum by Torsethaugen.

#### PM/ITTC/ISSC spectrum

The PM type spectra have the general form

\[
S(\omega) = \frac{A}{\omega^5} \exp\left(-\frac{B}{\omega^4}\right). \tag{7.11}
\]

They apply to fully developed sea states in open seas. The PM spectrum is given from wind speed \( V \) at 19.5 meters height

\[
A = 0.0081 g^2; \quad B = 0.74 (g/V)^4. \tag{7.12}
\]

The peak frequency of the spectrum, \( \omega_p \), may be used in place of the wind speed, giving

\[
A = 0.0081 g^2; \quad B = \frac{5}{4} \omega_p^4. \tag{7.13}
\]

The ITTC and ISSC spectra are identical for open seas, and given by the significant wave height \( H_s \) and peak wave frequency \( \omega_p \)

\[
A = 0.31 H_s^2 \omega_p^4; \quad B = 1.25 \omega_p^4. \tag{7.14}
\]

\( H_s \) and \( \omega_p \) are interdependent and must be chosen with care.

#### JONSWAP spectrum

The JONSWAP spectrum may according to Myrhaug [194] be formulated as

\[
S(\omega) = \frac{g^2}{\omega^5} \exp\left(-\frac{5}{4} \omega_p^4\right) \gamma \exp\left(-\frac{1}{2} (\frac{\omega}{\omega_p})^2\right), \tag{7.15}
\]

\( \alpha, \gamma \) and \( \sigma \) are parameters to be determined, and \( \omega_p \) is the peak frequency of the spectrum. \( \gamma \) is a peak parameter, and may vary between 1 and 7. For \( \alpha = 0.0081 \) and \( \gamma = 1 \), the JONSWAP spectrum is identical to the PM spectrum. The JONSWAP spectrum was developed after
measurements in an area of the North Sea which is relatively shallow and close to land. It is a spectrum for not fully developed seas, with a much sharper peak than the PM type spectra. Increasing $\gamma$ gives a sharper peak of the spectrum. Faltinsen [66] recommends the following values for the JONSWAP parameters

$$\gamma = 3.3, \quad \sigma = \begin{cases} 0.07 & \omega \leq \omega_p \\ 0.09 & \omega \geq \omega_p \end{cases}; \quad \alpha = 0.2 \frac{H_s^2 \omega_p^4}{g^2}. \tag{7.16}$$

To keep within the validity area of the spectrum, Myrhaug [194] suggests the following constriction on $H_s$ and $\omega_p$

$$1.25/\sqrt{H_s} < \omega_p < 1.75/\sqrt{H_s}. \tag{7.17}$$

If no information on $\gamma$ is available, DNV proposes the following $\gamma$ based on $H_s$ and $\omega_p$:

$$k = \frac{2\pi}{\omega_p \sqrt{H_s}},$$

$$k \leq 3.6 \Rightarrow \gamma = 5,$$

$$k \leq 5.0 \Rightarrow \gamma = \epsilon(5.75-1.15k),$$

$$k > 5.0 \Rightarrow \gamma = 1. \tag{7.18}$$

**Doubly peaked (Torsethaugen) spectrum**  The spectra presented above may all be termed wind generated, and have been criticized for not giving a good representation of the low-frequency wave energy, or swell. Torsethaugen [314] developed a doubly peaked wave spectrum; a low frequency peak due to swell, and a high frequency peak due to locally wind generated waves. The spectrum was developed by curve fitting of experimental data from the North Sea, and was standardized under Norsok Standard [207]. All parameters are found by the significant wave height and spectral peak period. The total wave spectrum is a sum of two spectral peaks, called primary and secondary. For fully developed seas, the secondary peak vanishes, and the wave energy is centered in a narrow peak about the spectral peak period $T_p = 2\pi/\omega_p$. For the same significant wave height $H_s$, primary peak periods higher than the value for fully developed seas can only be due to swell components, with a secondary peak of locally wind generated waves. For lower primary peak periods, the total energy cannot be set up by local winds alone, and an additional secondary swell peak is needed. For $T_p > 6.6H_s$, the primary peak is in the swell generated, and for $T_p < 6.6H_s$, the primary peak is in the wind generated. When the sea state is dominated by swell, the shape of the wave specter will differ significantly from the JONSWAP and PM wave spectra. The formulation of the doubly peaked spectrum is not presented here, but it is implemented in the Marine Systems Simulator [195] GNC Toolbox as $[\omega, S] = \text{torset}(H_s, \omega_p, \omega_{\max}, N)$, where $\omega_{\max}$ is the maximum frequency and $N$ the number of wave components.

**Comparison of the spectra**  Figure 7.6 shows a comparison of the PM, ITTC and JONSWAP spectra, all for $H_s = 6$ and $\omega_p = 1.5/\sqrt{H_s} \approx 0.61$. The PM spectrum does not account for the actual wave height, and contains less energy than the two others. The ITTC and JONSWAP spectra contains approximately the same amount of energy, but the JONSWAP spectrum has more of this energy close to $\omega_p$. Figure 7.7 shows how the doubly peaked spectrum varies with the peak frequency for constant $H_s$. The two spectra with thick lines are examples of a primary swell spectrum (largest peak for low frequencies) and a primary wind-generated spectrum.
Figure 7.6: Comparison of PM, ITTC and JONSWAP spectra.

Figure 7.7: Doubly peaked wave spectrum for varying $\omega_p$ and constant $H_s$. 
Linear wave theory limitations  Linear wave theory only considers non-breaking waves. A rule-of-thumb for modelling of non-breaking waves is

\[
\frac{h}{\lambda} \leq 1/7, \\
\downarrow \\
\lambda \geq 7h,
\]  

(7.19)

where \(h\) is the wave height (peak-to-peak) and \(\lambda\) the wave length. \(\lambda\) is related to the wave number \(k\) by

\[
k = \frac{1}{\lambda},
\]

(7.20)

In deep water, the dispersion relation is

\[
k = \frac{\omega^2}{g},
\]

(7.21)

where \(\omega\) is the wave frequency and \(g\) the acceleration of gravity. This gives

\[
\omega = \sqrt{kg} = \sqrt{\frac{g}{\lambda}} \leq \sqrt{\frac{g}{7h}} \approx \sqrt{\frac{1.4}{h}}.
\]

In terms of the wave amplitude \(\zeta = h/2\), we get the maximum valid wave frequency from the wave amplitude

\[
\omega \leq \sqrt{\frac{g}{14\zeta}} \approx \sqrt{\frac{0.7}{\zeta}},
\]

or equivalently the maximum valid wave amplitude from the wave frequency

\[
\zeta \leq \frac{g}{14\omega^2} \approx \frac{0.7}{\omega^2}.
\]

Defining the wave length to height ratio as \(\mu\), the corresponding wave frequency can be calculated from \(h\) and \(\kappa\)

\[
\mu = \frac{\lambda}{h}, \\
\omega = \sqrt{\frac{g}{\mu h}}.
\]

Common spreading functions

The most commonly used spreading function is according to Myrhaug [194] formulated as

\[
D(\psi - \psi_0) = \begin{cases} 
\frac{2^{s-1}2!(s-1)!}{\pi(2s-1)!} \cos^{2s}(\psi - \psi_0) & \text{for } -\frac{\pi}{2} < (\psi - \psi_0) < \frac{\pi}{2}, \\
0 & \text{elsewhere}
\end{cases}
\]

(7.22)

\(s\) is here an integer, with \(s = 1\) recommended by ITTC and \(s = 2\) recommended by ISSC. Figure 7.8 shows the spreading function for varying values of \(s\). Increasing \(s\) centers the wave energy more about the mean direction \(\psi_0\).
Implementation examples

The following figures show how the number of waves can be decreased by the methods described above. The wave spectrum is composed of a Torsethaugen frequency spectrum with significant wave height $H_s = 4m$ and peak frequency $\omega_p = 1\text{rad/s}$, and a spreading function with mean wave direction $\psi_0 = -30^\circ$ and spreading factor $s = 4$. The number of frequencies $N$ and number of directions $M$ were kept constant at $N = 20$ and $M = 10$ for all realizations. Figure 7.9 shows the wave spectrum for frequency cutoff factor $\xi = 4$ and the direction limit $\psi_{\text{lim}} = 0$. Clearly, a lot of insignificant components are included for high frequencies and far from the mean wave direction. If instead choosing $\xi = 2.5$ and the direction limit $\psi_{\text{lim}} = 20^\circ$, a much better resolution in the interesting area (i.e. about the peak frequency and mean direction) is obtained, as can be seen in Figure 7.10. The next step is to reduce the number of wave components. Figure 7.11 shows the chosen wave components as red stars when choosing the wave component energy limit $\kappa = 0.005$. A total of 96 wave components is included and 104 discarded. If in addition choosing the frequencies and directions at random within each interval the result is shown in Figure 7.12. To justify the exclusion of insignificant wave components, examples of surface realizations with varying number of wave components are shown in Figures 7.13 to 7.15. Figure 7.13 shows a realization of the spectrum defined above with all 200 wave components included. Figure 7.14 shows a realization of the same spectrum with the 100 most important wave components and Figure 7.15 shows a realization of the same spectrum with the 50 most important wave components. The difference between the realizations with 200 and 100 components is minimal, and yet the computational effort for the latter is only half. The difference is noticeable for the realization with 50 components, but the major waves are clearly unaltered; the difference lies in the smaller ripples in the surface due to the smaller wave components.
Figure 7.9: Torsethaugen frequency spectrum with $H_s = 4m$ and $\omega_p = 1rad/s$, spreading function with $\psi_0 = -30^\circ$ and $s = 4$, number of frequencies and directions $N = 20$ and $M = 10$, frequency cutoff factor $\xi = 4$ and wave direction limit $\psi_{\text{lim}} = 0$.

Figure 7.10: Torsethaugen frequency spectrum with $H_s = 4m$ and $\omega_p = 1rad/s$, spreading function with $\psi_0 = -30^\circ$ and $s = 4$, number of frequencies and directions $N = 20$ and $M = 10$, frequency cutoff factor $\xi = 2.5$ and wave direction limit $\psi_{\text{lim}} = 20^\circ$. 
Figure 7.11: Wave spectrum with wave energy limit $\kappa = 0.005$. The frequencies and directions of the components, which are shown as red stars, are not chosen at random.

Figure 7.12: Wave spectrum with wave energy limit $\kappa = 0.005$. The frequencies and directions of the components, which are shown as red stars, are chosen at random.
Figure 7.13: Realization of a Torsethaugen spectrum with 200 wave components.

Figure 7.14: Realization of a Torsethaugen spectrum with 100 wave components.
Joint distribution for wind and waves

The parameters describing the wind and wave state are interdependent. Johannessen et al. [132] have developed a joint distribution for wind (described by the average wind speed 10 meters above sea level $\bar{U}_{10}$) and waves (described by significant wave height $H_s$ and peak period $T_p = 2\pi/\omega_p$) in the northern North Sea, based on measurements in the 1973-99 period. Here we present only the mean peak period and mean wind speed as function of significant wave height, for use as a rule of thumb in modelling

$$E(T_p) = 4.883 + 2.680H_s^{0.54},$$
$$E(\bar{U}_{10}) = 1.764 + 3.426H_s^{0.78}. \quad (7.23, 7.24)$$

7.1.2 Wind

Wind is commonly divided in two components; a mean value and a fluctuating component, or gust. The mean component decreases with the distance to the ground, whereas the gust is approximately constant with the distance to the ground. Wind is in reality a three-dimensional phenomenon, but the descriptions commonly used are restricted to velocities in the horizontal plane, parameterized by the velocity $U$ and the direction $\psi$.

**Mean wind component**

The mean velocity $\bar{U}$ at elevation $z$ may be written as

$$\frac{\bar{U}(z)}{\bar{U}_{10}} = \frac{5}{2} \sqrt{\kappa} \ln \frac{z}{z_0}; \quad z_0 = 10 \exp\left(-\frac{2}{5\sqrt{\kappa}}\right). \quad (7.25)$$

$\bar{U}_{10}$ is the 1 hour mean wind speed at 10m elevation and $\kappa$ is the sea surface drag coefficient (Myrhaug [194]). Slowly-varying variations in the mean wind velocity may be implemented by a 1st order Gauss-Markov Process (Fossen [80])

$$\dot{\bar{U}} + \mu \bar{U} = w. \quad (7.26)$$

$w$ is Gaussian white noise and $\mu \geq 0$ is a constant. For $\mu = 0$, this is a random walk process. The magnitude of the velocity should be restricted by saturation elements

$$0 \leq \bar{U}_{\min} \leq \bar{U} \leq \bar{U}_{\max}. \quad (7.27)$$

The variation in wind direction may be implemented in a similar way

$$\dot{\psi} + \mu_2 \psi = w_2 \quad (7.28)$$
$$\psi_{\min} \leq \psi \leq \psi_{\max}, \quad (7.29)$$

where $w_2$ and $\mu_2$ are white noise and a positive constant respectively. No saturation elements are necessary here, but may be implemented if wanted.
Wind gust

The wind gust is commonly described by a spectrum, in a similar way as for waves. A widely used formulation is the Harris wind spectrum

\[ S(f) = \frac{4\kappa L U_{10}}{(2 + f^2)^{\frac{5}{6}}} \]

\( L \) is a scaling length, \( \kappa \) is the sea surface drag coefficient and \( f \) is the frequency in Hz. Example values for the parameters are \( L = 1800m \) and \( \kappa = 0.0026 \). The Harris spectrum was based on measurements over land, and more recent studies have given alternative representations. Norsok Standard [207] recommends a wave spectrum which also varies with

\[ S(f) = 320 \left( \frac{U_{10}}{10} \right)^2 \left( \frac{f}{10} \right)^{0.45} \left( 1 + x^n \right)^{\frac{3}{4}} \]

\( x = 172 f \left( \frac{z}{10} \right)^{2/3} \left( \frac{U_{10}}{10} \right)^{-3/4} \).

Figures 7.16 shows the Harris and NORSOK wave spectra for \( U_{10} = 10m/s \). It is clear that the NORSOK spectrum contains more energy at lower frequencies.

A realization of the wind state may be done by superposing the gust realization and the mean wind at the desired elevation. The gust realization is done in the same manner as a realization of a sea state from a wave spectrum. Harmonic component number \( i \) is defined as

\[ U_{gi}(t) = \sqrt{2S(f_i)\Delta f_i} \cos(2\pi f_it + \phi_i), \]

where \( f_i \) is the frequency, \( \Delta f_i \) the frequency interval and \( \phi_i \) an evenly distributed phase angle. The total wind realization with \( N \) gust components is then written as

\[ U(z, t) = \bar{U}(z) + \sum_{i=1}^{N} U_{gi}(t). \]

Implementation example  Figure 7.17 shows a wind time series using the models defined above. The wind direction is slowly-varying with mean 0 degrees, the mean wind speed is 10 m/s and the wind gust is modelled by a NORSOK wind spectrum with 100 frequency components.

7.1.3 Water Current Model

We may divide current modelling in two levels of detail:

- Surface current, for use in modelling of surface vessel response
- Full current profile, for use in modelling of risers, anchor lines etc.

Surface current

For modelling of surface vessels or other applications in the proximity of the surface, a 2-dimensional current model is sufficient. If the current is given by magnitude \( V_c \) and direction in the NED frame \( \psi_c \), the current velocity vector \( \nu_c \) may be written as

\[ \nu_c = [V_c \cos(\psi_c), V_c \sin(\psi_c), 0]^T \]
Figure 7.15: Realization of a Torsethaugen spectrum with 50 wave components

Figure 7.16: Harris and NORSOK wind spectra.
The variation in current velocity may be implemented by a 1st order Gauss-Markov Process (Fossen [80])

\[
\dot{V}_c + \mu V_c = w.
\]

(7.35)

\(w\) is Gaussian white noise and \(\mu \geq 0\) is a constant. For \(\mu = 0\), this is a random walk process. The magnitude of the velocity should be restricted by saturation elements

\[
V_{c,\text{min}} \leq V_c \leq V_{c,\text{max}}.
\]

(7.36)

The variation in current direction may be implemented in a similar way

\[
\dot{\psi}_c + \mu_2 \psi_c = w_2
\]

\(\psi_{c,\text{min}} \leq \psi_c \leq \psi_{c,\text{max}}\).

(7.37)

In addition, the current is often divided in two components: Tidal and wind generated. If actual measurement data are available, these may be separated, but for most purposes they may be lumped into one component.

**Current profile**

In some applications, the actual current profile in which the current varies with the depth, is needed. If no actual field measurements are available, DNV recommends the following current
profile $V_c(z)$, with $z$ the depth (positive downwards)

$$V_c(z) = V_{c,tide}(z) + V_{c,wind}(z)$$

(7.39)

$$V_{c,tide}(z) = V_{c,tide}\left(\frac{h - z}{h}\right)^{1/7} \text{ for } z \geq 0$$

(7.40)

$$V_{c,wind}(z) = V_{c,wind}\left(\frac{h_0 - z}{h_0}\right)^{1/7} \text{ for } 0 \leq z \leq h_0$$

(7.41)

$$V_{wind}(z) = 0 \text{ for } z \geq h_0,$$

(7.42)

where $V_{c,tide}$ is the tidal current velocity at surface level, $V_{c,wind}$ is the wind generated current velocity at surface level, $h$ is the water depth, and $h_0$ is the reference depth for wind generated current, example value $h_0 = 50m$.

The wind generated current may be taken as $V_{c,wind} = 0.015U_{10}$, where $U_{10}$ is the mean wind velocity 10 meters above sea level. In addition, the current profile should be stretched or compressed vertically to account for the change in water depth with the surface elevation due to waves. This may however be neglected at large water depths.

### 7.2 Kinematics

#### 7.2.1 Reference Frames

The different reference frames used are illustrated in the Figures 7.18 - 7.19, and are described below:

- The Earth-fixed reference frame is denoted as the $X_EY_EZ_E$-frame. Measurement of the vessel’s position and orientation coordinates are done in this frame relatively to a defined origin. One should notice that each position reference system has its own local coordinate system, which has to be transformed into the common Earth-fixed reference frame.

- In sea keeping analysis (vessel motions in waves) the hydrodynamic frame $X_hY_hZ_h$-frame is generally moving along the path of the vessel with the $x$-axis positive forwards, $y$-axis positive to the starboard, and $z$-axis positive downwards. The $X_hY_h$-plane is assumed fixed and parallel to the mean water surface. The vessel is assumed to oscillate with small amplitudes about this frame such that linear theory may apply for modelling of the perturbations. Often in forward speed sea keeping analysis the hydrodynamic frame is moving forward with constant vessel speed $U$. In station keeping operations (dynamic positioning) about the coordinates $x_d$, $y_d$, and $\psi_d$ the hydrodynamic frame is Earth-fixed and denoted as the reference-parallel frame $X_RY_RZ_R$. It is rotated to the desired heading angle $\psi_d$, and the origin is translated to the desired $x_d$ and $y_d$ position coordinates for the particular station keeping operation studied. Assuming small amplitudes of motion it is convenient to use this frame in the development of the control schemes.

- The body-fixed $XYZ$-frame is fixed to the vessel body with the $x$-axis positive forwards, $y$-axis positive to the starboard and $z$-axis positive downwards. For ships it is common to assume that the centre of gravity is located in the centre line of the vessel, and that the submerged part of the vessel is symmetric about the $xz$- plane (port/starboard). Here it is assumed that the origin is located in the mean oscillatory position (flotation point) in the average water plane. Hence, the centre of gravity is then located at $(x_G, 0, z_G)$ in body coordinates. The motion and the loads acting on the vessel are calculated in this frame.
The vectors defining the generalized vessel’s Earth-fixed position and orientation, and the body-fixed translation and rotation velocities are using SNAME [268] notation given by

\[
\begin{align*}
\eta_1 &= [x, y, z]^T, \\
\eta_2 &= [\phi, \theta, \psi]^T, \\
\nu_1 &= [u, v, w]^T, \\
\nu_2 &= [p, q, r]^T.
\end{align*}
\]

(7.43)

Here, \(\eta_1\) denotes the position vector in the Earth-fixed frame, and \(\eta_2\) is a vector of Euler angles. \(\nu_1\) denotes the body-fixed linear surge, sway and heave velocity vector, and \(\nu_2\) denotes the body-fixed angular roll, pitch and yaw velocity vector. For surface vessels the orientation is normally represented in terms of Euler angles (Fossen [78]).

**Definition 7.1** Degrees-of-Freedom - DOF. For a general body, the DOF is the set of independent displacements and rotations that completely specify the displaced position and orientation of the craft. A body that can move freely in the 3D space has maximum 6 DOFs with three translational and three rotational components.

**Definition 7.2** Configuration Space. The \(n\)-dimensional configuration space is the space of possible positions and orientations that a body may attain subject to external constraints.

Ships and rigs are often described in the horizontal plane only with surge sway and yaw (with \(n = 3\) DOFs), while underwater vehicles may be described in \(n = 6\) DOFs including both the horizontal (surge, sway, yaw) plane and vertical plane (heave, roll, pitch)

**Definition 7.3** Working Space The working space is a reduced space of dimension \(m < n\) in which the control objective is defined.

An underactuated vehicle has independent control forces and moments in only some DOFs such that \(r < n\). For an underwater vehicle such as a Remotely Operated Vehicle (ROV) that is self-stabilized in roll and pitch, the working space may be \(m = 4\). If the number of actuators \(r\) are less than \(m\), the the control problem is referred to as underactuated control. Stabilizing and tracking controllers for underactuated vehicles are usually designed by considering a working space of dimension \(m < n\) satisfying \(m = r\) (fully actuated in the working space but not in the configuration space). This may be the case for the ROV. Notice that many ships and underwater vehicles are overactuated with \(r > m\) making the control allocation to an optimization problem with infinite many solutions.

### 7.2.2 The Euler Angle Transformation

**Linear velocity**

The Euler angles result from a sequence of rotations which is not arbitrary. The Euler angles appear from the following rotation sequence:

- Let \(X_3Y_3Z_3\) be a Earth-fixed reference frame that is translated such that the origin is coinciding with the body-fixed \(XYZ\)-frame.

- Rotate \(X_3Y_3Z_3\) a yaw angle \(\psi\) about the \(Z_3\)-axis such that a new reference frame denoted as \(X_2Y_2Z_2\) appears.
Figure 7.18: Definition of surge, sway, heave, roll, pitch and yaw modes of motion in body-fixed frame.

Figure 7.19: Definition of frames: Earth-fixed, reference-parallel and body-fixed.
• Rotate $X_2 Y_2 Z_2$ a pitch angle $\theta$ about the $Y_2$-axis such that a new reference frame denoted as $X_1 Y_1 Z_1$ appears.

• Rotate $X_1 Y_1 Z_1$ a roll angle $\phi$ about the $X_1$-axis such that the reference frame denoted as $XYZ$ appears.

The rotation sequence (Fossen [78]) is then given by

$$\mathbf{J}_1(\eta_2) = \mathbf{C}_{z,\psi}^T \mathbf{C}_{y,\theta}^T \mathbf{C}_{x,\phi}^T, \quad (7.44)$$

where

$$\begin{align*}
\mathbf{C}_{z,\psi} &= \begin{bmatrix}
c\psi & s\psi & 0 \\
-s\psi & c\psi & 0 \\
0 & 0 & 1
\end{bmatrix}, \\
\mathbf{C}_{y,\theta} &= \begin{bmatrix}
c\theta & 0 & -s\theta \\
0 & 1 & 0 \\
s\theta & 0 & c\theta
\end{bmatrix}, \\
\mathbf{C}_{x,\phi} &= \begin{bmatrix}
1 & 0 & 0 \\
0 & c\phi & s\phi \\
0 & -s\phi & c\phi
\end{bmatrix}.
\end{align*} \quad (7.45)$$

Hence, the rotation matrix $\mathbf{J}_1(\eta_2) \in \text{SO}(3)$ can be defined as

$$\mathbf{J}_1(\eta_2) = \begin{bmatrix}
c\phi c\theta & -s\phi c\psi + c\psi s\phi & s\psi c\phi + c\psi c\phi s\theta \\
-c\psi c\phi - s\psi c\theta & s\psi s\phi & c\psi c\phi s\theta \\
-s\phi c\theta & c\psi s\phi & c\theta s\phi
\end{bmatrix}, \quad (7.48)$$

where $c \cdot = \cos(\cdot)$, $s \cdot = \sin(\cdot)$ and $t \cdot = \tan(\cdot)$. The symbol $\text{SO}(3)$ denotes special orthogonal group of order 3. Thus, the linear velocities of the vessel in the Earth-fixed frame is given by the transformation

$$\dot{\eta}_1 = \mathbf{J}_1(\eta_2) \mathbf{\nu}_1. \quad (7.49)$$

**Remark 7.1** Two vectors $\mathbf{x}_1, \mathbf{x}_2 \in \mathbb{R}^n$ are said to be orthogonal if $\mathbf{x}_1^T \mathbf{x}_2 = \mathbf{x}_2^T \mathbf{x}_1 = 0$. A set of vectors $\mathbf{x}_i \in \mathbb{R}^n, i = 1, 2, \ldots, m$, is said to be orthonormal if

$$\mathbf{x}_i^T \mathbf{x}_j = \begin{cases} 0 & \text{if } i \neq j, \\ 1 & \text{if } i = j. \end{cases} \quad (7.50)$$

**Remark 7.2** The reference frame matrix $\mathbf{C}_{i,\alpha} \in \text{SO}(3)$ has the property (Fossen [78])

$$\mathbf{C}_{i,\alpha}^T \mathbf{C}_{i,\alpha} = \mathbf{C}_{i,\alpha}^T \mathbf{C}_{i,\alpha} = \mathbf{I}, \quad (7.51)$$

where $\mathbf{C}_{i,\alpha}$ denotes a rotation angle $\alpha$ about the $i$-axis, and where

$$\det \mathbf{C}_{i,\alpha} = 1, \quad (7.52)$$

implies that $\mathbf{C}_{i,\alpha} = [\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3]$ and $\mathbf{C}_{i,\alpha}^T = [\mathbf{c}_1^T, \mathbf{c}_2^T, \mathbf{c}_3^T]^T$ consist of orthonormal columns with the consequence that

$$\mathbf{C}_{i,\alpha}^{-1} = \mathbf{C}_{i,\alpha}^T. \quad (7.53)$$
Remark 7.3 Based on (7.51) we can derive that
\[ J^{-1}_1(\eta_2) = J^T_1(\eta_2). \] (7.54)

Hence,
\[ \nu_1 = J^{-1}_1(\eta_2)\dot{\eta}_1 = J^T_1(\eta_2)\dot{\eta}_1. \] (7.55)

Angular velocity rotation

The angular velocities of the vessel in the Earth-fixed frame is given by
\[ \dot{\eta}_2 = J_2(\eta_2)\nu_2. \] (7.56)

The angular velocities in the body-fixed frame appear from the relation (Fossen [78])
\[ \nu_2 = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + C_{x,\phi} \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + C_{x,\phi}C_{y,\theta} \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} = J_2^{-1}(\eta_2)\dot{\eta}_2, \] (7.57)

where we can expand (7.57) and derive that
\[ J_2^{-1}(\eta_2) = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & c\theta s\phi \\ 0 & -s\phi & c\theta c\phi \end{bmatrix}. \] (7.58)

Inverting (7.58) gives \( J_2(\eta_2) \in \mathbb{R}^{3 \times 3} \) according to
\[ J_2(\eta_2) = \begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi/c\theta & c\phi/c\theta \end{bmatrix}, \quad c\theta \neq 0, \] (7.59)

where \( c \cdot = \cos(\cdot) \), \( s \cdot = \sin(\cdot) \) and \( t \cdot = \tan(\cdot) \).

6 DOF kinematics

The linear and angular velocities of the vessel in the Earth-fixed frame is given by
\[ \dot{\eta} = \begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0_{3 \times 3} \\ 0_{3 \times 3} & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix} = J(\eta_2)\nu. \] (7.60)

6 DOF kinematics (small roll and pitch angle representation)

A frequently used kinematic approximation for metacentric stable vessels with small roll and pitch angles is
\[ \dot{\eta} = J(\eta_2)\nu, \] (7.61)
\[ \eta_2 = [0, 0, \psi]^T, \] (7.62)

where
\[ J(\psi) = \begin{bmatrix} C_{z,\psi}^T & 0_{3 \times 3} \\ 0_{3 \times 3} & I_{3 \times 3} \end{bmatrix}. \] (7.63)

Remark 7.4 For small \( \phi \)-roll and \( \theta \)-pitch angles (less that 10°), we approximate \( \cos \phi \approx 1 \) and \( \cos \theta \approx 1 \) and \( \sin \phi \approx 0 \) and \( \sin \theta \approx 0 \).
3 DOF kinematics (horizontal motion)

In many practical applications only the horizontal modes of motion are of interest. It is then convenient to find an appropriate kinematic representation only for the horizontal-plane motion. For surge, sway and yaw the 6 DOF kinematics reduces to

$$\dot{\eta} = R(\psi)\nu, \quad R^{-1}(\psi) = R^T(\psi)$$

(7.64)

where we have redefined the state vectors according to $\eta = [x \ y \ \psi]^T$ and $\nu = [u \ v \ r]^T$, and

$$R(\psi) = C^T_{z,\psi} = \begin{bmatrix} c\psi & -s\psi & 0 \\ s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$ 

In a reference-parallel formulation we can define

$$\dot{\eta}_R = R(\psi - \psi_d)\nu.$$ 

(7.65)

**Remark 7.5** In hydrodynamic literature several conventions for hydrodynamic coefficients and reference frames are used. In Faltinsen [66], added mass, wave radiation damping and restoring coefficients are denoted as $A_{ij}$, $B_{ij}$, $C_{ji}$ with the $x$-axis positive backwards, $y$-axis positive to the starboard and $z$-axis positive upwards. In Newman [199] a second convention is used with the $x$-axis positive forwards, $y$-axis positive upwards and $z$-axis positive to the starboard. Here, the notation of SNMNE [268] is used with the $x$-axis positive forwards, $y$-axis positive to the starboard and $z$-axis positive downwards. The SNMNE notation is more established in hydrodynamic and control problems related to manoeuvring and positioning of ships.

**Remark 7.6** Integration of the angular velocities $\int_0^t \nu_2(\tau) d\tau$ does not have any physical meaning. In order to have proper generalized coordinates $\dot{\eta}_2$ should be integrated.

**Remark 7.7** $J_2(\eta_2)$ is undefined for $\theta = \pm90^\circ$. Consequently $J_2^{-1}(\eta_2) \neq J_2^T(\eta_2)$ due to the singularity in $\theta = \pm90^\circ$. For surface vessels this is not of any problem. However, for underwater vehicles special precaution must be taken. Then a so-called quaternion (Euler parameters) could be used, see Fossen [78] for details.

**Remark 7.8** For low speed manoeuvring and change of setpoint in station keeping operations, the $X_RY_RZ_R$-frame will be time-varying following the trajectories for $x_d(t)$, $y_d(t)$ and $\psi_d(t)$.

**Remark 7.9** In the reference-parallel frame $X_RY_RZ_R$ (and hydrodynamic frame as well) assuming small amplitudes of motion in yaw ($|\psi - \psi_d| \approx 0$), we have

$$\dot{\eta}_R = R(\psi - \psi_d)\nu \approx I_{3 \times 3}\nu,$$

(7.66)

where $I_{3 \times 3} \in \mathbb{R}^{3 \times 3}$ is the identity matrix. Thus, the reference-parallel frame is convenient for design of linear controllers as we avoid the nonlinear kinematics.

**Remark 7.10** When considering moored structures, such as turret-moored tankers or moored semi-submersibles, it is common to locate the origin of the Earth-fixed frame in the natural equilibrium point for the mooring system. This position is often referred to as the field zero point (FZP). The body-fixed frame is located in the geometrical centre of the mooring system on the structure; for turret-moored ships this will be the centre of turret (COT).
Low-frequency (LF) and wave-frequency (WF) motion

Figure 7.20: The total motion of a ship is modeled as a LF response with the WF response added as an output disturbance.

### 7.3 Vessel Dynamics

In mathematical modeling of the marine vessel dynamics it is common to separate the total model into a low-frequency (LF) model and a wave-frequency (WF) model by superposition. Hence, the total motion is a sum of the corresponding LF and the WF components, see Figure 7.20. The WF motions are assumed to be caused by first-order wave loads. Assuming small amplitudes these motions will be well represented by a linear model. The LF motions are assumed to be caused by second-order mean and slowly varying wave loads, current loads, wind loads, mooring and thrust forces. These motions are generally nonlinear, but linear approximations about certain operating points can be found.

One should notice that for surface vessels, only the horizontal motions i.e. the surge, sway and yaw degrees-of-freedom (DOF), are subject for control. In the design of positioning control systems one should expect that it is sufficient to only consider the horizontal-plane dynamics in the LF model. This is normally an appropriate assumption, where the effect of vertical-plane dynamics i.e. the heave, roll and pitch DOF will be of minor practical interest for the control problem. However, according to Sørensen and Strand [290] under certain circumstances this assumption may cause unacceptable reduction in the control performance. In their work it was suggested to characterize the control problem into two categories whether the natural periods in roll and pitch are within or outside the bandwidth of the positioning controller. Hence, the modelling problem can either be regarded as a three or a six DOF problem, where the following prerequisites can be made:

- **3 DOF**: For conventional ship and catamaran hulls in the low-frequency model only the three horizontal-plane surge, sway and yaw DOF are of practical interest for the controller design. For those vessels it can be assumed that the LF vertical-plane dynamics and the thruster action will not have any mutual influence on each other.
• **6 DOF**: For marine structures with a small-waterplane-area and low metacentric height, which results in relatively low hydrostatic restoring compared to the inertia forces, an unintentional coupling phenomenon between the vertical and the horizontal planes through the thruster action can be invoked. Examples are found in semi-submersibles and SWATHs, which typically have natural periods in roll and pitch in the range of $35-65$ seconds. If the inherent vertical damping properties are small, the amplitudes of roll and pitch may be emphasized by the thruster’s induction by up to $2^\circ-5^\circ$ in the resonance range. These oscillations have caused discomfort in the vessel’s crew and have in some cases limited the operation. Hence, both the horizontal and vertical planes DOF should be considered.

Concerning the WF model it is normal to include all 6 DOF no matter how the LF model is formulated. The vertical-plane WF motions (heave, roll and pitch) must be used to adjust the acquired position measurements to some defined origin on the vessel. This could be in the plane of the undisturbed free surface with the z-axis through the centre of gravity (CG), the centre of buoyancy (CB) or the flotiation point. Thus, the raw position reference system measurements will be strongly influenced by the vertical-plane motions depending on the locations of the installed GPS antennas, hydroacoustic position reference system transducers, Artemis receivers, Taut wire, etc. relative to the defined origin.

### 7.3.1 Nonlinear Low-Frequency Vessel Model

The nonlinear 6 DOF body-fixed coupled equations of the LF motions in surge, sway, heave, roll, pitch and yaw are written as follows

$$
M\ddot{\nu} + C_{RB}(\nu)\dot{\nu} + C_A(\nu,\tau)\nu + D(\kappa,\nu) + G(\eta) = \tau_{\text{env}} + \tau_{\text{moor}} + \tau_{\text{ice}} + \tau_{\text{thr}}. \quad (7.67)
$$

The right-hand expression of (7.67) represents generalized external forces acting on the vessel and is treated later in this section. Forces in surge, sway and heave and moments in roll, pitch and yaw are referred to as generalized forces. $\tau_{\text{env}} \in \mathbb{R}^6$ represents the slowly-varying environmental loads with the exception of current loads acting on the vessel. The effect of current is already included on the left hand side of (7.67) by the introduction of the relative velocity vector. $\tau_{\text{thr}} \in \mathbb{R}^6$ represents the generalized forces generated by the propulsion system. Even if only the horizontal-plane surge, sway and yaw DOF are subject for control, geometrical coupling to the vertical-plane heave, roll and pitch DOF, will be invoked due to the actual locations of the thrusters. As discussed above the produced thrust components in the vertical-plane will be important to consider for marine structures with small-waterplane-area. If the vessel is attached to a mooring system, the effect of this is represented by $\tau_{\text{moor}} \in \mathbb{R}^6$. For operation in ice the corresponding loads from level ice, ice floes and ice ridges are modeled by $\tau_{\text{ice}} \in \mathbb{R}^6$. In Nguyen et al. [204] models of level ice loads are presented.
Generalized inertial forces, $M\ddot{\nu}$:

The system inertia matrix $M \in \mathbb{R}^{6 \times 6}$ including added mass is defined as

$$
M = \begin{bmatrix}
    m - X_\dot{u} & 0 & -X_\dot{w} & 0 & mz_G - X_\dot{q} & 0 \\
    0 & m - Y_\dot{\phi} & 0 & -mz_G - Y_\dot{\phi} & 0 & m_x G - Y_\dot{\phi} \\
    -Z_\dot{u} & 0 & m - Z_\dot{w} & 0 & -mz_G - Z_\dot{q} & 0 \\
    0 & m_\dot{z}_G - M_\dot{u} & 0 & I_x - K_\dot{\phi} & 0 & -I_{xx} - K_\dot{\phi} \\
    0 & m_x G - N_\dot{\phi} & 0 & 0 & I_y - M_\dot{q} & 0 \\
    0 & m_x G - N_\dot{\phi} & 0 & 0 & 0 & I_z - N_\dot{\phi}
\end{bmatrix},
$$

(7.68)

where $m$ is the vessel mass, $I_x$, $I_y$ and $I_z$ are the moments of inertia about the $x$-, $y$- and $z$-axes and $I_{xx} = I_{yy}$ are the products of inertia. The zero-frequency added mass coefficients $X_\dot{u}, X_\dot{w}, X_\dot{q}, Y_\dot{\phi}$, and so on at low speed in surge, sway, heave, roll, pitch and yaw due to accelerations along the corresponding and the coupled axes are defined as in Faltinsen [66]. Hence, it can be shown that the system inertia matrix is symmetrical and positive definite (Appendix A), i.e. $M = M^T > 0$ and $M = 0$.

Generalized Coriolis and centripetal forces, $C_{RB}(\nu)\nu + C_A(\nu_t)\nu_t$:

The matrix $C_{RB}(\nu) \in \mathbb{R}^{6 \times 6}$ is the skew-symmetric Coriolis and centripetal matrix of the rigid body written (Fossen [78])

$$
C_{RB}(\nu) = \begin{bmatrix}
    0 & 0 & 0 & c_{41} & -c_{51} & -c_{61} \\
    0 & 0 & 0 & -c_{42} & c_{52} & -c_{62} \\
    0 & 0 & 0 & -c_{43} & -c_{53} & c_{63} \\
    -c_{41} & c_{42} & c_{43} & 0 & -c_{54} & -c_{64} \\
    c_{51} & -c_{52} & c_{53} & c_{54} & 0 & -c_{65} \\
    c_{61} & c_{62} & -c_{63} & c_{64} & c_{65} & 0
\end{bmatrix},
$$

(7.69)

where

$$
c_{41} = m z_G \nu, \quad c_{42} = m w, \quad c_{43} = m (z_G \nu - v), \quad c_{44} = m (z_G \nu + v),
\quad c_{45} = m (z_G \nu + u), \quad c_{46} = I_{xx} \nu - I_x \nu.
\quad (7.70)
$$

Wichers [326] divided the effect of current into two parts: the potential part and the viscous part. The Coriolis and centripetal matrix of the added mass including the potential part of the current load is formulated according to

$$
C_A(\nu_t) = \begin{bmatrix}
    0 & 0 & 0 & 0 & -c_{a51} & -c_{a61} \\
    0 & 0 & 0 & -c_{a42} & 0 & -c_{a62} \\
    0 & 0 & 0 & -c_{a43} & -c_{a53} & 0 \\
    0 & c_{a42} & c_{a43} & 0 & -c_{a54} & -c_{a64} \\
    c_{a51} & c_{a53} & c_{a54} & 0 & -c_{a65} \\
    c_{a61} & c_{a62} & 0 & c_{a64} & c_{a65} & 0
\end{bmatrix},
$$

(7.71)
where

\[
\begin{align*}
\alpha_{42} &= -Z_{\dot{w}}w - X_{\dot{u}}u_r - X_q q \\
\alpha_{43} &= \dot{Y}_{\dot{p}}p + \dot{Y}_{\dot{v}}v_r + \dot{Y}_{\dot{r}}r \\
\alpha_{51} &= Z_{\dot{q}}q + Z_{\dot{w}}w + X_{\dot{u}}u_r \\
\alpha_{53} &= -X_{\dot{q}}q - X_{\dot{u}}u_r - X_{\dot{w}}w \\
\alpha_{61} &= -Y_{\dot{v}}v_r - Y_{\dot{p}}p - Y_{\dot{r}}r \\
\alpha_{62} &= X_{\dot{u}}u_r + X_{\dot{w}}w + X_{\dot{q}}q \\
\alpha_{64} &= X_{\dot{q}}q + X_{\dot{w}}w + M_{\dot{q}}q
\end{align*}
\]  

(7.72)

Notice that the so-called Munk moments appear from the expression \( C_A(\nu_r)\nu_r \), see Faltinsen [66] and Newman [199].

**Generalized damping and current forces, \( D(\kappa, \nu_r) \):**

The damping vector may be divided into a nonlinear and a linear component according to

\[
D(\kappa, \nu_r) = D_L(\kappa, \nu_r)\nu_r + d_{NL}(\nu_r, \gamma_r). 
\]  

(7.73)

The linear damping is assumed to vanish for increasing speed as the flow becomes turbulent. In order to incorporate this effect the linear damping is multiplied with an an exponential decaying functions according to:

\[
D_L(\kappa, \nu_r) = \begin{bmatrix}
X_{u_r} e^{-\kappa |u_r|} & \cdots & X_{r_r} e^{-\kappa |r|} \\
\vdots & \ddots & \vdots \\
N_{u_r} e^{-\kappa |u_r|} & \cdots & N_{r_r} e^{-\kappa |r|}
\end{bmatrix},
\]

where \( \kappa \) is a positive constant such that \( \kappa \in \mathbb{R}^+ \).

Furthermore, the effect of current load is normally included in the nonlinear damping term by the definition of the relative velocity vector according to:

\[
\nu_r = \begin{bmatrix}
\dot{u} - u_c & \dot{v} - v_c & \dot{w} & \dot{p} & \dot{q} & \dot{r}
\end{bmatrix}^T.
\]

(7.74)

The horizontal current components in surge and sway are defined as:

\[
\begin{align*}
u_c &= V_c \cos (\beta_c - \psi), \\
v_c &= V_c \sin (\beta_c - \psi),
\end{align*}
\]

(7.75)

where \( V_c \) and \( \beta_c \) are the current velocity and direction respectively, see Figure 7.19. Notice that the current velocity components in heave, roll, pitch and yaw are not considered. The total relative current velocity is then defined as for \( \dot{u}_r = \dot{u} - u_c \), and \( \dot{v}_r = \dot{v} - v_c \) according to

\[
U_{cr} = \sqrt{\dot{u}_r^2 + \dot{v}_r^2}.
\]

(7.76)

The relative drag angle is found from the following relation:

\[
\gamma_r = \text{atan2}(-v_r, -u_r),
\]

(7.77)

where \( \text{atan2} \) is the four quadrant arctangent function of the real parts of the elements of \( X \) and \( Y \), such that \(-\pi \leq \text{atan2}(Y, X) \leq \pi \). The nonlinear damping is assumed to be caused by turbulent skin friction and viscous eddy-making, also denoted as vortex shedding, Faltinsen [66] and Faltinsen and Sortland [67].
Nonlinear damping and current forces  Assuming small vertical motions, the 6-dimensional nonlinear damping vector is often formulated as:

\[ \mathbf{d}_{NL}(v_r, \gamma_r) = 0.5 \rho_w L_{pp} \begin{bmatrix} DC_{cx}(\gamma_r)|U_{cr}|U_{cr} \\ DC_{cy}(\gamma_r)|U_{cr}|U_{cr} \\ BC_{cz}(\gamma_r)|w|w \\ B^2 C_{cy}(\gamma_r)|p|p + z_{py} DC_{cy}(\gamma_r)|U_{cr}|U_{cr} \\ L_{pp} BC_{cb}(\gamma_r)|q|q - z_{pz} DC_{cz}(\gamma_r)|U_{cr}|U_{cr} \\ L_{pp} DC_{cy}(\gamma_r)|U_{cr}|U_{cr} \end{bmatrix}, \]  

(7.78)

where \( C_{cx}(\gamma_r), C_{cy}(\gamma_r), C_{cz}(\gamma_r), C_{cb}(\gamma_r) \) and \( C_{cy}(\gamma_r) \) are the nondimensional drag coefficients found by model tests for the particular vessel with some defined location of the origin. \( B \) is the breadth, \( \rho_w \) is the density of water, \( L_{pp} \) is the length between the perpendiculars, and \( D \) is the draft. The second contributions to roll and pitch are the moments caused by the nonlinear damping and current forces in surge and sway, respectively, attacking in the corresponding centers of pressure located at \( z_{py} \) and \( z_{pz} \).

For relative current angles \( |\beta_c - \psi| >> 0 \) the cross flow principle (Faltinsen [66]) may be applied to calculate the nonlinear current loads in sway \( d_2 \) and yaw \( d_6 \) in (7.78). In sway the cross flow formulation is written

\[ d_2 = 0.5 \rho_w \int_{L_{pp}} D(x) C_{cy}^{2D}(\gamma_r, x) (v_r + rx) |v_r + rx| dx, \]  

(7.79)

where \( D(x) \) is the longitudinal varying draft, and \( C_{cy}^{2D}(\gamma_r, x) \) is the 2 dimensional drag coefficient. Similar as for sway, the cross flow expression in yaw is written

\[ d_6 = 0.5 \rho_w \int_{L_{pp}} D(x) C_{cy}^{2D}(\gamma_r, x) (v_r + rx) |v_r + rx| dx. \]  

(7.80)

For simplicity let us assume that \( D(x) = D \), and that the drag coefficient is constant over the entire vessel length, that is \( C_{cy}^{2D}(\gamma_r, x) = C_{cy}^{2D} \). Typical \( C_{cy}^{2D} \) values for ship (varying between 0.45 – 1.4 dependent on Reynolds number), see Figure 6.16 in Faltinsen [66]. As shown in Figure 6.20 on page 197 in Faltinsen [66] for slender bodies the drag coefficient may be adjusted for 3-dimensional effects due to the vertical vortex system at ship ends.

Let the origin be located in the middle of the ship, such that the distance to the bow is \( L_{pp}/2 \) and to the stern is \(-L_{pp}/2\). Hence, a solution of (7.79) may be found to be

\[ d_2 = 0.5 \rho_w D C_{cy}^{2D} \int_{L_{pp}} (v_r + rx) |v_r + rx| dx = \]

\[ 0.5 \rho_w D C_{cy}^{2D} \left( \int_0^{L_{pp}/2} (\text{sgn}(v_r)|v_r^2 + rv_r x| + \text{sgn}(x)|rv_r x + r^2 x^2|) dx + \right) = \]

\[ 0.5 \rho_w D C_{cy}^{2D} \left( \begin{bmatrix} \frac{L_{pp}}{2} v_r^2 + \frac{L_{pp}^2}{8} v_r r \\ \text{sgn}(v_r) \left[ \frac{L_{pp}}{8} v_r^2 + \frac{L_{pp}^2}{24} v_r r \right] \end{bmatrix} + \right) \]

(7.81)
sgn is the sign function defined such that \( \text{sgn}(x) = 1 \) for \( x \geq 0 \) and \( \text{sgn}(x) = -1 \) for \( x < 0 \). Notice that \( \text{sgn}(rx) = \text{sgn}(r) \text{sgn}(x) \).

Similarly, (7.80) is rewritten

\[
d_6 = 0.5 \rho_w D c_{2D} \int_{L_{pp}} x (v_r + rx) |v_r + rx| dx = 0.5 \rho_w D c_{2D} \left( \int_{L_{pp}/2}^{L_{pp}} \left( \text{sgn}(v_r, x) |v_r^2 x + rv_r x^2| + \text{sgn}(rx^2) |rv_r x^2 + r^2 x^3| \right) dx + \int_{-L_{pp}/2}^{0} \left( \text{sgn}(v_r, x) |x v_r^2 + rv_r x^2| + \text{sgn}(rx^2) |rv_r x^2 + r^2 x^3| \right) dx \right) = 0.5 \rho_w D c_{2D} \left( \text{sgn}(v_r) \left[ \frac{L_{pp}^2 v_r^2}{8} + \frac{L_{pp}^3 v_r}{24} \right] - \left| -\frac{L_{pp}^2 v_r^2}{8} + \frac{L_{pp}^3 v_r}{24} \right| + \text{sgn}(r) \left[ \frac{L_{pp}^2 v_r}{24} + \frac{L_{pp}^3 v_r}{44} x^2 + \frac{L_{pp}^3 v_r}{44} x^2 - \frac{L_{pp}^3 v_r}{64} x^2 \right] \right). \quad (7.82)
\]

Notice that \( \text{sgn}(rx^2) = \text{sgn}(r) \text{sgn}(x^2) = \text{sgn}(r) \).

Remark 7.11 One should notice that for sway and yaw in (7.78), it is assumed that

\[
0.5 \rho_w \int_{L_{pp}} D(x) C_{2D}^{c_{2D}} (\gamma_t, x) (v_r + rx) |v_r + rx| dx \approx 0.5 \rho_w L_{pp} D c_{2D} (\gamma_t) |U_r| U_c, 
\]

and

\[
0.5 \rho_w \int_{L_{pp}} D(x) C_{2D}^{c_{2D}} (\gamma_t, x) x (v_r + rx) |v_r + rx| dx \approx 0.5 \rho_w L_{pp}^2 D c_{cy} (\gamma_t) |U_r| U_c, 
\]

implying that the effect of yaw angular velocity, \( r \), is small relative to the effect of \( v_r \).

Remark 7.12 Furthermore, if the drag coefficient in sway is given by model tests, an alternative expression for the 2-dimensional drag coefficient applied in (7.79) and (7.80) may be found to be

\[
C_{2D}^{c_{2D}} = C_{cy} (\pi/2). \quad (7.83)
\]

Linear damping It is important to notice that for velocities close to zero, linear damping becomes more significant than the nonlinear damping. The strictly positive linear damping matrix \( D_L \in \mathbb{R}^{6 \times 6} \) caused by linear wave drift damping and the laminar skin friction is written as

\[
D_L = \begin{bmatrix}
X_u & 0 & X_w & 0 & X_q & 0 \\
0 & Y_u & 0 & Y_p & 0 & Y_r \\
Z_u & 0 & Z_w & 0 & Z_q & 0 \\
0 & K_u & 0 & K_p & 0 & K_r \\
M_u & 0 & M_w & 0 & M_q & 0 \\
0 & N_u & 0 & N_p & 0 & N_r \\
\end{bmatrix}. \quad (7.84)
\]

This kind of damping must not be confused with the frequency-dependent wave radiation damping used in the wave-frequency model. In general, the effect of wave radiation damping can be neglected in the LF model due to the low frequency of oscillation, especially for the horizontal modes of motions. For the vertical-plane modes (heave, roll and pitch), potential damping may be of interest in the frequency range of interest for control, and should therefore
be considered to be included in the LF model. Wave drift damping in surge can be interpreted as added resistance for the vessel advancing in waves and is proportional to the square of the significant wave height.

It is for most bodies hard to calculate the damping coefficients. A combination of empirical formulas, model tests and computational fluid dynamics (CFD) are normally used to find the damping coefficients. In Table 7.1 the dominating damping effects are described for the horizontal-plane DOF. The heave, roll and pitch will normally follow the same tendency as the sway and yaw.

The Keulegan-Carpenter number in Table 7.1 is defined as \( KC = UT/D \), where \( U \) is the free stream velocity, \( T \) is the oscillation period and \( D \) is the characteristic length of the body. For sway and yaw the nonlinear damping due to eddy-making will dominate the damping until very low velocity, where the damping behavior is seen to be linear, suggesting that laminar skin friction is present. In the literature some authors do not include any linear damping terms in the models. Instead they let the drag coefficients increase to large values for small velocities \( u_r, v_r, r \to 0 \). However, from a control point of view it is appropriate to include both linear and nonlinear damping, as this seems to be more in agreement with the physics.

The coefficients can be calculated by special software or found by model tests. In the last years the importance of wave-drift damping has been more appreciated, and some effort has been made to include wave-drift damping predictions in hydrodynamic software programs, see Finne and Grue [75] and the references therein. The cross-flow principle and strip theory can be used to calculate viscous damping in sway and yaw. The cross-flow principle assumes that the flow separates due to the cross-flow past the ship, and that the transverse forces on a cross section is mainly due to separated flow effects on the pressure distribution around the ship. The method is semi-empirical in the sense that empirical drag coefficients are employed. For conventional ships data on nonlinear drag coefficients for the horizontal-plane modes are available, see OCIMF [213]. Instead of applying semi-empirical methods, model tests are often used. When using model tests it should be kept in mind that scale effects may be important for the viscous forces. The transition from laminar to turbulent boundary layer is dependent on the Reynolds number, which is different in model and in full scale. This may also affect the separation point and thus

### Table 7.1: Dominating damping effects.

<table>
<thead>
<tr>
<th>Dominating damping</th>
<th>High sea state</th>
<th>Low sea state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>Linear wave drift. Nonlinear turbulent skin friction.</td>
<td>Nonlinear turbulent skin friction, when (</td>
</tr>
<tr>
<td>Sway</td>
<td>Nonlinear eddy-making. Linear wave drift.</td>
<td>Nonlinear eddy-making, when (</td>
</tr>
<tr>
<td>Yaw</td>
<td>Nonlinear eddy-making. Linear wave drift.</td>
<td>Nonlinear eddy-making, when (</td>
</tr>
</tbody>
</table>
the eddy-making damping. In cases with a clearly defined separation point, such as a bilge keel, scale effects are not supposed to be significant for the eddy-making damping. Wave drift damping is considered to be unaffected by scale effects.

If there already exists data from other ships, based on model tests or numerical calculations, experience has shown that using proper scaling techniques, these coefficients may represent a reasonable good estimate for ships with similar geometrical hull shapes. For further details about damping the reader is referred to Faltinsen [66], Faltinsen and Sortland [67], and Newman [199].

**Remark 7.13** In implementation of the linear damping terms for ships due to linear skin friction, it is suggested to ramp down these damping contributions when the relative speed exceeds 0.25 m/s. Notice that the linear damping due to wave drift should be maintained.

**Generalized restoring forces, G(η):**

Here it is assumed small roll and pitch angles, such that the restoring vector can be linearized to $G\eta$, where $G \in \mathbb{R}^{6 \times 6}$ is a matrix of linear generalized gravitation and buoyancy force coefficients and is for $xz$-plane symmetry written

$$G = -\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z_z & 0 & Z_\theta & 0 & 0 \\
0 & 0 & 0 & K_\phi & 0 & 0 \\
0 & M_z & 0 & M_\theta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

(7.85)

where the coefficients are defined as

$$Z_z \triangleq -\rho_w g A_{WP},$$

(7.86)

$$Z_\theta = M_z \triangleq \rho_w g \int_A x \, dA,$$

(7.87)

$$K_\phi \triangleq -\rho_w g \nabla (z_G - z_B) - \rho_w g \int_{A_{WP}} y^2 \, dA = -\rho_w g V G M_T,$$

(7.88)

$$M_\theta \triangleq -\rho_w g \nabla (z_G - z_B) - \rho_w g \int_{A_{WP}} x^2 \, dA = -\rho_w g V G M_L.$$  

(7.89)

Here, $g$ is the acceleration of gravity, $A_{WP}$ is the waterplane area, $dA = dxdy$, $\nabla$ is the displaced volume of water, and $G M_T$ and $G M_L$ are the transverse and longitudinal metacentric heights, respectively.

### 7.3.2 Environmental Loads

The slowly-varying environmental loads acting on the vessel are composed of

$$\tau_{\text{env}} = \tau_{\text{wind}} + \tau_{\text{wave}}.$$  

(7.90)

Remember that the current load vector already is included by the relative velocity vector in the nonlinear damping term.
Wind load model

The effect of wind may be divided into mean, slowly-varying and rapidly-varying wind loads. The relative wind velocity vector is defined as

\[ \mathbf{v}_{rw} = [ u - u_w \ v - v_w \ w \ p \ q \ r ]^T. \]  

The components of the wind velocities are defined according to

\[ u_w = V_w \cos (\beta_w - \psi), \quad v_w = V_w \sin (\beta_w - \psi), \]  

where \( V_w \) and \( \beta_w \) are the wind velocity and direction respectively, see Figure 7.19. The total relative wind velocity is then defined as for \( \mathbf{v}_{rw} = u - u_w, \) and \( \mathbf{v}_{rw} = v - v_w \) according to

\[ U_{wr} = \sqrt{u_{rw}^2 + v_{rw}^2}. \]  

The relative wind angle is found from the following relation:

\[ \gamma_w = \tan^{-1}(-v_{rw}, u_{rw}). \]  

The wind load vector is then formulated

\[ \tau_{\text{wind}} = 0.5 \rho_a \begin{bmatrix} A_x C_{wx}(\gamma_w) |U_{wr}| U_{wr} \\ A_y C_{wy}(\gamma_w) |U_{wr}| U_{wr} \\ 0 \\ A_y L_{yz} C_{wy}(\gamma_w) |U_{wr}| U_{wr} \\ -A_x L_{xz} C_{wx}(\gamma_w) |U_{wr}| U_{wr} \\ A_y L_{xy} C_{wy}(\gamma_w) |U_{wr}| U_{wr} \end{bmatrix}. \]  

Here, \( \rho_a \) is the density of air, \( L_{oa} \) is the overall length of the vessel, \( L_{xz} \) and \( L_{yz} \) are the vertical distances between transverse and longitudinal origin and the wind load point of attack, \( A_x \) and \( A_y \) are the lateral and longitudinal areas of the non-submerged part of the ship projected on the \( xz \)-plane and \( yz \)-plane. \( C_{wx}(\gamma_w), C_{wy}(\gamma_w), \) and \( C_{wy}(\gamma_w) \) are the non-dimensional wind coefficients in surge, sway and yaw respectively. These coefficients are often found by model testing or by employing semi-empirical formulas as presented in Isherwood [128] and Blendermann [32].

**Remark 7.14** In station keeping operations the wind velocity is often assumed to be much larger than the vessel velocity, such that the relative wind angle may be simplified to \( \gamma_w = \beta_w - \psi, \) and \( U_{wr} = \sqrt{u_{rw}^2 + v_{rw}^2}. \)

Wave load model

The wave drift loads contribute to a significant part of the total excitation force in the low-frequency model. The second-order wave effects are divided into mean, slowly varying (difference frequencies) and rapidly varying (sum frequencies) wave loads. For the applications considered here the effect of the rapidly varying second order wave loads can be neglected. The determination of the second-order wave effects can be done by means of quadratic transfer functions (Newman [198] and Faltinsen [66])

\[ \tau_{\text{wave}2}^i = \tau_{\text{wave}}^i + \tau_{\text{wave}2}^i, \quad i = 1..6 \]

\[ = \sum_{j=1}^{N} \sum_{k=1}^{N} A_j A_k \left[ T_{jk}^e \cos ((\omega_k - \omega_j) t + \varepsilon_k - \varepsilon_j) + T_{jk}^s \sin ((\omega_k - \omega_j) t + \varepsilon_k - \varepsilon_j) \right], \]  

\[ = \sum_{j=1}^{N} \sum_{k=1}^{N} A_j A_k \left[ T_{jk}^e \cos ((\omega_k - \omega_j) t + \varepsilon_k - \varepsilon_j) + T_{jk}^s \sin ((\omega_k - \omega_j) t + \varepsilon_k - \varepsilon_j) \right], \]  

\[ \]
where $\omega_j$ is the wave frequency, $A_j$ is the wave amplitude and $\varepsilon_j$ is a random phase angle. The superscript $c$ and $s$ denote $\cos$ and $\sin$, respectively. The quadratic transfer functions $T_{jk}$ are dependent on both the first and second order velocity potentials, which require a nonlinear panel methodology. In addition, it is time-consuming to calculate the $T_{jk}$ for all combinations of $\omega_k$ and $\omega_j$. This motivates to derive some simplifications. One should notice that the transfer functions when $k = j$, $T_{jj}$, represents the mean wave loads, and can be calculated from the first order velocity potential only. The most interesting slowly-varying wave loads are those where $\omega_k - \omega_j$ is small and the loads are truly slowly-varying. Normally, $T_{jk}$ will not vary significantly with the frequency. Then, the following approximation by Newman [198] will give satisfactory results

$$T^{ic}_{jk} = T^{ic}_{kj} = \frac{1}{2} (T^{ic}_{jj} + T^{ic}_{kk})$$

$$T^{is}_{jk} = T^{is}_{kj} = 0$$

The slowly-varying loads are approximated by the mean drift loads, and hence, the computation becomes much simpler and less time consuming. This approximation based on frequency dependent wave drift coefficients will then further be applied. By dividing the sea wave spectrum (usually of Pierson-Moskowitz type) into $N$ equal frequency intervals with corresponding wave frequency, $\omega_j$, and amplitude, $A_j$, the wave drift loads are found to be

$$\tau_{wave2}^i = \tau_{wm}^i + \tau_{Ws}^i$$

$$= 2 \left( \sum_{j=1}^{N} A_j (T_{jj}^i (\omega_j, \beta_{wave} - \psi))^{1/2} \cos (\omega_j t + \varepsilon_j) \right)^2,$$

where $T_{jj}^i > 0$ is the frequency-dependent wave drift function and $\beta_{wave}$ is the mean wave direction (assumed to follow the same sign convention as wind and current). A disadvantage with this approximation is the numerical generation of high-frequency components of no physical meaning. By numerical filtering this can be avoided. Eq. (7.99) can also be extended to include wave spreading. In general, the second-order wave loads are much smaller than the first-order wave loads. The second-order wave loads are proportional to the square of the wave amplitude, whereas the first-order wave loads are proportional to the wave amplitude. This means that the second-order wave loads have an increased importance for increasing sea states.

### 7.3.3 Linear Wave-Frequency Model

In linear theory small waves and amplitudes of motion are assumed. The WF motion is in the literature calculated in the hydrodynamic frame. The hydrodynamic problem of a vessel in regular waves is solved as two sub-problems wave reaction and wave excitation, which are added together (Faltinsen, [66]). Potential theory is assumed, neglecting viscous effects.

- **Wave Reaction**: Forces and moments on the vessel when the vessel is forced to oscillate with the wave excitation frequency. The hydrodynamic loads are identified as added mass and wave radiation damping terms.

- **Wave Excitation**: Forces and moments on the vessel when the vessel is restrained from oscillating and there are incident waves. This gives the wave excitation loads which are
composed of so-called Froude-Kriloff (forces and moments due to the undisturbed pressure field as if the vessel was not present) and diffraction forces and moments (forces and moments because the presence of the vessel changes the pressure field).

In station keeping operations assuming small motions about the coordinates $x_d$, $y_d$, and $\psi_d$, the coupled equations of WF motions can in the hydrodynamic frame be formulated according to

\[
\begin{align*}
M(\omega)\ddot{\eta}_{Rw} + D_p(\omega)\dot{\eta}_{Rw} + G\eta_{Rw} &= \tau_{wave1} \\
J(\bar{\eta}_2)\dot{\eta}_{Rw} &= \tau_{wave1},
\end{align*}
\]

where $\eta_{Rw} \in \mathbb{R}^6$ is the WF motion vector in the hydrodynamic frame, $\eta_w \in \mathbb{R}^6$ is the WF motion vector in the Earth-fixed frame, and $\bar{\eta}_2 = [0 \quad 0 \quad \psi_d]^T$. $\tau_{wave1} \in \mathbb{R}^6$ is the first order wave excitation vector, which is dependent on the vessel heading relative to the incident wave direction. $M(\omega) \in \mathbb{R}^{6 \times 6}$ is the system inertia matrix containing frequency dependent added mass coefficients in addition to the vessel’s mass and moment of inertia. $D_p(\omega) \in \mathbb{R}^{6 \times 6}$ is the wave radiation (potential) damping matrix. $G \in \mathbb{R}^{6 \times 6}$ is the linearized restoring coefficient matrix due to the gravity and buoyancy affecting heave, roll and pitch only. It is assumed that the mooring lines will not affect the WF motion (Triantafyllou, [316]).

Generally, a time domain equation cannot be expressed with frequency domain coefficient. However, this is a common used formulation denoted as a pseudo-differential equation. An important feature of the added mass terms and the wave radiation damping terms is the memory effects, which in particular are important to consider for non-stationary cases, e.g. rapid changes of heading angle. Memory effects can be taken into account by introducing a convolution integral or a so-called retardation function (Newman, [199]) or state space models as suggested by Kristiansen and Egeland [155] and Fossen [81].

Results from model tests and computer programs for vessel response analysis often come in the form of transfer functions or tables of coefficients. This applies to linear wave-induced motions, 2nd-order wave drift and slowly varying motions. To a large extent, linear theory is sufficient for describing wave-induced motions and loads on vessels. This is especially true for moderate sea states. In irregular seas, the response of the vessel may be calculated by adding results for regular waves of different amplitudes, frequencies and directions. Nonlinear effects become increasingly important in severe sea states, and is still a subject for further research.

### 7.3.4 Thrust Servo Model

Usually, perfect control action without imposing the thruster dynamics are assumed. Unfortunately, this will not be true in a real system. The thrust response is affected by the dynamics in the actuators and the drive system. This will cause reduced command following capabilities such as phase lag and amplitude reduction when the frequency increases. There will also be a loss of thrust efficiency due to disturbances in the water inflow to the thruster blades, caused by thrust-to-thrust and thrust-to-hull interactions, current and vessel velocities. In addition the influence from the free surface will affect the thrust efficiency. Reduced thruster efficiency caused by disturbances in the water inflow is compensated for in the controller. This will be addressed in detail in Chapter 9. However, experience obtained from full-scale experiments indicates that a first-order model is well suited as a first approximation. Hence, the thruster dynamics is represented by the following simplified model:

\[
\dot{\tau}_{thr} = -A_{thr}(\tau_{thr} + \tau_c),
\]
where \( \tau_c \in \mathbb{R}^3 \) is the commanded thrust vector in surge, sway and yaw produced by the controller. \( A_{\text{thr}} = \text{diag}\{1/T_1, 1/T_2, 1/T_3\} \) is the diagonal thruster dynamics matrix. Determination of the commanded thrust vector will be treated more in detail in Section 8.3.

### 7.4 Static Analysis of Cable Segments

#### 7.4.1 Basic Assumptions

A basic assumption when working with cables is that we have no bending stiffness and no torsional stiffness. This means that only axial stiffness will be considered, and this simplifies the analysis of cables considerable compared to for instance beams.

Another assumption is that the axial tension in the cable is small enough to allow us to operate in the linear range of stress/strain relationship. This is reasonable for metallic cables and synthetic cables under normal tension.

![Figure 7.21: An infinitesimal cable segment.](image)

Consider Figure 7.21 which shows a cable segment. The cable’s weight per unit length in air is denoted \( w_a = mg \), where \( g \) is the earth’s gravity and \( m \) is the mass per length unit. When the cable is submerged a hydrostatic force will appear according to

\[
B = \rho_w g A. \tag{7.102}
\]

\( \rho_w \) is the density of water and \( A \) is the cross-sectional area. Given a cable tension, \( T_c \), Triantafyllou [315] shows that the effective tension, \( T \), may be written as

\[
T = T_c + p_e A, \tag{7.103}
\]

where \( p_e \) is the hydrostatic pressure at the specific point of the cable. This leads to the following definition of the stretched cable’s weight in water, \( w_1 \):

\[
w_1 = w_a - B. \tag{7.104}
\]

When working with cables we assume that the material is isotropic. Isotropic materials’ properties are by definition independent of direction. Such materials have only 2 independent variables (i.e. elastic constants) in their stiffness and compliance matrices. The generalized Hooke’s law for axial strain may be written as

\[
\sigma = E \varepsilon = \frac{T}{A}, \tag{7.105}
\]

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where $\sigma$ is strain, $E$ is Young’s modulus, $\varepsilon$ is axial strain and $\nu$ is Poisson’s ratio. Applies this law on the submerged cable and find

$$
\varepsilon = \frac{1}{E} \left( \frac{T_c}{A} + 2\nu p_c \right).
$$

(7.106)

Notice that a Poisson’s ratio of $\frac{1}{2}$ results in an incompressible material. Poisson’s ratio for synthetic cables are close to $\frac{1}{2}$, while metallic materials have a ratio approximately $\frac{1}{3}$ (Triantafyllou [315]). A substantial simplification of the forthcoming analysis is achieved if we assume that Poisson’s ratio is close to $\frac{1}{2}$. Using (7.103) in (7.106) a simplified expression for the axial strain is found to be

$$
\varepsilon = \frac{1}{EA}(T_c + p_c A) = \frac{T}{EA}.
$$

(7.107)

This leads to the last assumption; the cross-sectional area of the cable, $A$, will not undergo significant changes due to the axial deformation of the cable.

### 7.4.2 Catenary Equations

Catenary equations are widely used in mooring analysis of anchored bodies. In mooring analysis the effect of each mooring line is first analyzed separately and thereafter summarized to constitute the complete mooring system, often consisting of several deployed mooring lines.

For anchored floating vessels it is assumed that each mooring line is fastened in the seabed and in the vessel. Vertical and horizontal forces caused by the mooring system will act on the vessel at the terminal points. However, often only the horizontal forces are important for the vessel response in the horizontal plane. Along the mooring line gravity and buoyancy forces are assumed to be most important. For this purpose a two-dimensional approach is often considered to be sufficient. If varying current in space is considered, a three-dimensional approach may be needed.

#### Two-dimensional approach

An infinite small element of a cable is shown in Figure 7.21. The tangential and normal forces to this element may be written as

$$
\mathbf{F} = \begin{bmatrix} F_t \\ F_n \end{bmatrix} = \begin{bmatrix} -T - w_1 \sin \varphi \ dp + (T + dT) \cos (d\varphi) \\ -w_1 \cos \varphi \ dp + (T + dT) \sin (d\varphi) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.
$$

(7.108)

Taking the limit

$$
\lim_{dp \to 0} \mathbf{F} \Rightarrow \begin{bmatrix} \frac{dT}{dp} \\ T \frac{dp}{d\varphi} \end{bmatrix} = \begin{bmatrix} w_1 \sin \varphi \\ w_1 \cos \varphi \end{bmatrix}.
$$

(7.109)

because

$$
\lim_{d\varphi \to 0} \begin{bmatrix} \sin (d\varphi) \\ \cos (d\varphi) \end{bmatrix} = \begin{bmatrix} d\varphi \\ 1 \end{bmatrix}.
$$

(7.110)

From the Figure 7.21 we also find that

$$
\begin{bmatrix} dx \\ dz \end{bmatrix} = \begin{bmatrix} dp \cos \varphi \\ dp \sin \varphi \end{bmatrix}.
$$

(7.111)
It is appropriate to formulate the equations for the tension as a function of the cable’s Lagrangian coordinate, denoted as $s$, along the unstretched length. The cable’s strain is given in (7.107), but may also be written as

$$\varepsilon = \frac{dp - ds}{ds} = \frac{dp}{ds} - 1,$$  \hspace{1cm} (7.112)

where $p$ is the cable’s stretched length. Triantafyllou [315] shows the relation between mass, buoyancy and specific weight of the stretched and unstretched cable are

$$m_0 ds = m dp, \hspace{1cm} (7.113a)$$
$$B_0 ds = B dp, \hspace{1cm} (7.113b)$$
$$w_z ds = w_1 dp. \hspace{1cm} (7.113c)$$

Combining (7.109) and (7.111) with (7.112) and (7.113) leads to the following two-dimensional catenary equations

$$\frac{dT}{ds} = w_z \sin \varphi, \hspace{1cm} (7.114a)$$
$$T \frac{d\varphi}{ds} = w_z \cos \varphi, \hspace{1cm} (7.114b)$$
$$\frac{dx}{ds} = (1 + \varepsilon) \cos \varphi, \hspace{1cm} (7.114c)$$
$$\frac{dz}{ds} = (1 + \varepsilon) \sin \varphi. \hspace{1cm} (7.114d)$$

Notice that (7.114) is a set of ODEs which may be written in the form (10.1) described in Section 10.1.1. If we know that an axial force is acting at the end of the cable, this is an initial value problem where $s$ is the independent variable. Numerical routines like MATLAB’s ode45 could be utilized, but in the next section we will show that there exists an analytical solution.

**Solution of the two-dimensional equations**

Consider the cable in Figure 7.22. An axial force $T_B$ acts in point B, and no vertical force is present in point A. This is decomposed into a vertical and a horizontal force

$$V_B = T_B \sin \varphi, \hspace{1cm} (7.115a)$$
$$H_B = T_B \cos \varphi, \hspace{1cm} (7.115b)$$
$$\tan \varphi = \frac{V_B}{H_B}. \hspace{1cm} (7.115c)$$

Combining (7.114a) and (7.114b) gives

$$\frac{dT}{T} = \tan \varphi \, d\varphi. \hspace{1cm} (7.116)$$

Integrating and applying the boundary condition in (7.115b) leads to

$$T = \frac{H_B}{\cos \varphi}. \hspace{1cm} (7.117)$$
Figure 7.22: A marine mooring cable.

Introducing (7.117) into (7.114b)
\[ \frac{d\varphi}{ds} = \frac{w}{H_B} \cos^2 \varphi. \]  
(7.118)

Integration and use of (7.115c) leads to
\[ \tan \varphi = \frac{V_B}{H_B} - \frac{w}{H_B} (L - s), \]  
(7.119)

where \( L \) is the cable’s unstretched length.

\( \varphi \) may now easily be calculated as a function of \( s \) in (7.119). First, notice the following relations
\[ \cos x = \frac{1}{\pm \sqrt{1 + \tan^2 x}}, \]  
(7.120)
\[ \sinh^{-1}(x) = \ln(x + \sqrt{1 + x^2}), \]  
(7.121)
\[ \sin x = \frac{\tan x}{\pm \sqrt{1 + \tan^2 x}}. \]  
(7.122)

Use of (7.120) leads to an expression for the tension along the Lagrangian variable \( s \)
\[ T(s) = \frac{H_B}{\cos \varphi} = H_B \sqrt{1 + \tan^2 \varphi}, \]  
(7.123)
\[ = H_B \sqrt{1 + \left( \frac{V_B - w_z(L - s)}{H_B} \right)^2} = \sqrt{H_B^2 + \left( \frac{V_B - w_z(L - s)}{H_B} \right)^2}. \]

Notice that for \( s = L \) this gives the relation \( T_B^2 = H_B^2 + V_B^2 \).

Sometimes it may be valuable to calculate the Cartesian coordinates of the line. Eqs. (7.114c) and (7.114d) can be rewritten to:
\[ \frac{dx}{ds} = (1 + \varepsilon) \cos \varphi = \left( 1 + \frac{T(s)}{EA} \right) \cos \varphi, \]  
(7.124a)
\[ \frac{dz}{ds} = (1 + \varepsilon) \sin \varphi = \left( 1 + \frac{T(s)}{EA} \right) \sin \varphi. \]  
(7.124b)
Still assuming constant $H_B$ we find that
\[
\frac{dx}{ds} = \left( 1 + \frac{T(s)}{EA} \right) \cos \varphi = \cos \varphi + \frac{H_B}{EA}, \tag{7.125}
\]
\[
\frac{dz}{ds} = \left( 1 + \frac{T(s)}{EA} \right) \sin \varphi = \sin \varphi + \frac{V(s)}{EA} = \sin \varphi + \frac{V_B - w_z(L - s)}{EA}. \tag{7.126}
\]

Use of (7.120) and (7.122) yields
\[
\int_0^s dx = \int_0^s \cos \varphi + \frac{H_B}{EA} ds = \int_0^s \frac{1}{\sqrt{1 + \left( \frac{V_B - w_z(L - s)}{H_B} \right)^2}} + \frac{H_B}{EA} ds, \tag{7.127}
\]
\[
\int_0^s dz = \int_0^s \sin \varphi + \frac{V_B - w_z(L - s)}{EA} ds = \int_0^s \frac{V_B - w_z(L - s)}{\sqrt{1 + \left( \frac{V_B - w_z(L - s)}{H_B} \right)^2}} + \frac{V_B - w_z(L - s)}{EA} ds, \tag{7.128}
\]
\[
\downarrow
\]
\[
\int_0^s dx = \int_0^s \frac{H_B}{\sqrt{H_B^2 + (V_B - w_z(L - s))^2}} + \frac{H_B}{EA} ds, \tag{7.129}
\]
\[
\int_0^s dz = \int_0^s \frac{V_B - w_z(L - s)}{\sqrt{H_B^2 + (V_B - w_z(L - s))^2}} + \frac{V_B - w_z(L - s)}{EA} ds. \tag{7.130}
\]

Integration of this expression along the variable $s$ leads to
\[
x(s) = \frac{H_B}{w_z} \ln \left[ V_B - w_z(L - s) + \sqrt{H_B^2 + (V_B - w_z(L - s))^2} \right] - \frac{H_B}{w_z} \ln \left[ V_B - w_zL + \sqrt{H_B^2 + (V_B - w_zL)^2} \right] + \frac{H_B s}{EA} + C_x, \tag{7.131}
\]
\[
z(s) = \frac{1}{w_z} \left( \sqrt{H_B^2 + (V_B - w_z(L - s))^2} - \sqrt{H_B^2 + (V_B - w_zL)^2} \right) + \frac{1}{EA} \left( V_B s + \frac{w_z}{2}(L - s)^2 \right) + C_z. \tag{7.132}
\]

We may want our coordinates for the start point $s = 0$ to be $(0,0)$. Applies this boundary condition to find $C_x$ and $C_z$. This gives
\[
C_x = 0, \tag{7.133}
\]
\[
C_z = -\frac{1}{2EA} w_z L^2. \tag{7.134}
\]
In some literature (e.g. Triantafyllou [315]) it is common to write \( x(s) \) as a hyperbolic sine. If we make use of (7.121) we may write

\[
x(s) = \frac{H_B}{w_z} \ln \left[ \frac{V_B - w_z(L - s)}{H_B} + \sqrt{1 + \frac{(V_B - w_z(L - s))^2}{H_B^2}} \right] - \frac{H_B}{w_z} \ln \left[ \frac{V_B - w_zL}{H_B} + \sqrt{1 + \frac{(V_B - w_zL)^2}{H_B^2}} \right] + \frac{H_B s}{EA}.
\]  

(7.135)

\[
x(s) = \frac{H_B}{w_z} \left[ \sinh^{-1} \left( \frac{V_B - w_z(L - s)}{H_B} \right) - \sinh^{-1} \left( \frac{V_B - w_zL}{H_B} \right) \right] + \frac{H_B s}{EA}.
\]  

(7.136)

Sometimes it may be useful to express the \( z \)- and \( x \)-coordinates as a function of each other. Remember that \( V(0) = 0 \). Eq. (7.123) may be written as

\[
T(s) = \sqrt{H_B^2 + (w_z s)^2}.
\]  

(7.137)

Eq. (7.119) may be rewritten to

\[
\tan \varphi = \frac{V(s)}{H_B} = \frac{dx}{dz} = \frac{w_z s}{H_B}.
\]  

(7.138)

Combining the equations above leads to

\[
T = \sqrt{H_B^2 + H_B^2 \left( \frac{dz}{dx} \right)^2} = H_B \sqrt{1 + \left( \frac{dz}{dx} \right)^2}.
\]  

(7.139)

From Figure 7.21

\[
dp^2 = dx^2 + dz^2,
\]  

(7.140)

and by use of the relation \( dp = ds(1 + \varepsilon) \), we find

\[
\frac{ds}{dx} = \frac{1}{1 + \varepsilon} \sqrt{1 + \left( \frac{dz}{dx} \right)^2},
\]  

(7.141)

\[
\Rightarrow s(x) = \int_0^x \frac{1}{1 + \varepsilon} \sqrt{1 + \left( \frac{dz}{dx} \right)^2} \, dx.
\]  

(7.142)

Combining with (7.138) the following relation appears

\[
\frac{dz}{dx} = \frac{w_z}{H_B} \int_0^x \frac{1}{1 + \varepsilon} \sqrt{1 + \left( \frac{dz}{dx} \right)^2} \, dx.
\]  

(7.143)

If we assume *inelastic* material and substitute

\[
\sinh u = \frac{dz}{dx},
\]  

(7.144)

which leads to

\[
\sinh u = \frac{w_z}{H_B} \int_0^x \sqrt{1 + \sinh^2 u} \, dx,
\]  

(7.145)

\[
\sinh u = \frac{w_z}{H_B} \int_0^x \cosh u \, dx.
\]  

(7.146)
Derivation on both sides

\[
\frac{du}{dx} \cosh u = \frac{w_z}{H_B} \cosh u,
\]
(7.147)

\[
\frac{du}{dx} = \frac{w_z}{H_B} dx,
\]
(7.148)

\[
u = \frac{w_z}{H_B} x + C_1.
\]
(7.149)

Considering the boundary condition \(\sinh u|_{x=0} = 0\), we find that \(C_1 = 0\). Integration of (7.144)

\[
z = \frac{w_z}{H_B} \cosh \frac{w_z}{H_B} x + C_2.
\]
(7.150)

Boundary condition \(z|_{x=0} = 0\) \(\Rightarrow C_2 = -\frac{w_z}{H_B}\). Finally we have for the inelastic case:

\[
z(x) = \frac{w}{H_B} \cosh \frac{w}{H_B} x - \frac{w}{H_B}.
\]
(7.151)

For the elastic case the substitution in (7.144) does not solve the problem, and the final expression for \(z(x)\) becomes very unhandy.

**Catenary in the three-dimensional case**

This section is mainly based on Sagatun [239]. Remember the assumption of no significant reduction of the cable’s cross-sectional area when it is stretched. Considering the cable with one-dimensional strain along the Lagrangian variable \(s\), Hooke’s law is

\[
\sigma = \frac{T}{A} = E \varepsilon,
\]
(7.152)

\[
\Rightarrow T = \sigma A = EA \varepsilon,
\]
(7.153)

where \(\varepsilon\) is given in (7.112). Combining the equations we find that

\[
T = EA \left( \frac{dp}{ds} - 1 \right).
\]
(7.154)

Seeking a solution in Cartesian coordinates of the cable as a function of \(s\), the identity \(\frac{dr}{ds} = \frac{dr}{dp} \frac{dp}{ds}\) will be useful. \(r = [x \ y \ z]^T\) contains the Cartesian coordinates as a function of \(s\) or \(p\). This leads to the relation

\[
\frac{dr}{ds} = \frac{dr}{dp} \left( \frac{T}{EA} + 1 \right).
\]
(7.155)

Consider the first segment of a cable with distributed vertical force, \(w_z\), a vertical concentrated force, \(f_{1,z}\) and axial tension \(T\) in the end point. Inspection of Figure 7.23 gives

\[
f_{0,z} = T \frac{dz}{dp}|_{s=s_1} + \tilde{f}_{1,z} + w_z(s_1 - s_0),
\]
(7.156)

for static equilibrium. \(s_i\) indicates the value of \(s\) in node \(i\), and \(\tilde{f}_{i,k}\) indicates the value of concentrated force in node \(i\) and Cartesian direction \(k\).
Figure 7.23: A cable segment with concentrated and distributed loads (Sagatun [239]).

Assume that the cable is terminated in point 0, and that concentrated forces, \( \bar{f}_i \), may act in discrete points (cable nodes). Combination of this assumption and (7.156) written in vectorial form gives the following equation for the forces acting in the cable’s terminating point:

\[
f_0 = T \frac{dr}{dp} \bigg|_{s=L} + \sum_{i=1}^{n} \bar{f}_i + \sum_{i=1}^{n} w_i (s_i - s_{i-1}),
\]

(7.157)

where \( \{ s \in [0, L] \rightarrow \mathbb{R}, k \in [1, n] \rightarrow \mathbb{N} \mid s \in [s_{k-1}, s_k] \} \). \( n \) is the number of cable segments and \( L \) is the cable’s unstretched length. \( w_i = [w_{ix} \ w_{iy} \ w_{iz}]^T \) indicates the constant distributed force vector in segment \( i \).

An investigation of Figure 7.24 may make it easier to keep the indexes of the variables right.

Figure 7.24: A cable of several segments with concentrated forces, \( \bar{f}_i \), and distributed forces, \( w_i \). Notice the indexes.

The cable must be divided into a new segment when one of the following conditions are fulfilled:

1. A concentrated force occurs
2. A change in distributed force occurs
3. A change in cross-sectional area occurs
4. A change in the cable’s Young’s modulus occurs

A constant distributed force along the cable leads to a simplification of (7.157)

\[ f_0 = T \frac{dr}{dp} \Big|_{p(s=L)} + \sum_{i=1}^{n} \bar{f}_i + wL, \]  
(7.158)

where \( w \) is the constant distributed force along the cable.

Sagatun [239] gives the following derivation of a differential equation for the cable. It is assumed that (7.158) holds. Reordering of (7.158) leads to

\[ T \frac{dr}{dp} \Big|_{p(s=L)} = f_0 - \sum_{i=1}^{n} \bar{f}_i - wL. \]  
(7.159)

We want to find the tension vector as a function of \( \sigma \). Assume that \( \sigma(p) \) may be found from (7.155). Then the following relation must hold

\[ T \frac{dr}{dp}(s) = f_0 - \sum_{i=1}^{k} \bar{f}_i - ws, \]  
(7.160)

where \( \{ s \in [0, L] \to \mathbb{R}, k \in [1, n] \to \mathbb{N} \mid s \in [s_{k-1}, s_k] \} \). We define

\[ T(p) = T \begin{bmatrix} \frac{dr}{dp} \\ \frac{dr}{dp} \\ \frac{dr}{dp} \end{bmatrix} = T \frac{dr}{dp}, \]  
(7.161)

\[ T^T T = T^2 \frac{dr}{dp} \frac{dr}{dp}, \]  
(7.162)

where \( T \in \mathbb{R}^3 \). The substitution

\[ f_k = f_0 - \sum_{i=1}^{k} \bar{f}_i, \]  
(7.163)

where \( \{ k \in [1, n] \to \mathbb{N}, s \in [0, L] \to \mathbb{R} \mid s \in [s_{k-1}, s_k] \} \), gives

\[ T(s) = \sqrt{(f_k - ws)^T (f_k - ws)}. \]  
(7.164)

Combining (7.163) and (7.160) gives

\[ T \frac{dr}{dp}(s) = f_0 - \sum_{i=1}^{k} \bar{f}_i - ws = f_k - ws. \]  
(7.165)

Use of (7.155)

\[ T \frac{dr}{dp} = \left( \frac{1}{EA} + \frac{1}{T} \right)^{-1} \frac{dr}{ds} = f_k - ws. \]  
(7.166)
Reordering leads to
\[ \frac{dr}{ds} = (f_k - ws) \left( \frac{1}{EA} + \frac{1}{\sqrt{(f_k - ws)^T (f_k - ws)}} \right). \] (7.167)

Compare this equation to (7.124) found in the previous section. Remember that \( T = (f_k - ws) \) represents the tension in vectorial form. The trigonometric terms in (7.124) is a way of decomposing \( T \) into the \( xz \)-plane. This shows the relation between the two-dimensional case solved by Triantafyllou [315] and the three-dimensional case solved by Sagatun [239].

Substituting (7.164) gives the final differential equation
\[ \frac{dr}{ds} = (f_k - ws) \left( \frac{1}{EA} + \frac{1}{\sqrt{(f_k - ws)^T (f_k - ws)}} \right). \] (7.168)

This is an ODE with \( s \) as the independent variable. This may be solved directly from the rewritten form
\[ r_k = \int_{s_{k-1}}^{s_k} (f_k - ws) \left( \frac{1}{EA} + \frac{1}{\sqrt{(f_k - ws)^T (f_k - ws)}} \right) ds, \] (7.169)

where \( k \) is the actual cable segment. \( r_k \) is an expression for the local solution within segment \( k \).

Numerical methods could have been utilized to solve the ODE/IVP, but Sagatun [239] gives the following solution to the integral
\[ r_k(s) = \frac{\alpha(s)}{\beta^2} (f_k \beta^2 - f_k \otimes w \otimes w - f_k \otimes (P(f_k \otimes w))) - w \frac{1}{\beta^2} \sqrt{(f_k - ws)^T (f_k - ws)} + \frac{1}{EA} \left( f_k s - \frac{1}{2} w s^2 \right) + C_i, \] (7.170)

where \( \otimes \) denotes component wise multiplication and
\[ \beta = \sqrt{w^T w} = ||w||_2, \] (7.171)
\[ \alpha = \ln \left( \beta s - f_k^T w \right) + \sqrt{(f_k - ws)^T (f_k - ws)} \], (7.172)
\[ P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}. \] (7.173)

Assume that we want segment no. 1 of the global solution \( r(s) \) to start in the origin. We also want to ensure continuity between the segments. The following conditions should be fulfilled
\[ r(0) = 0, \] (7.174)
\[ r(s_i)^- = r(s_i)^+, \quad \{ i \in [1, n-1] \to \mathbb{N} \}. \] (7.175)

The integration constants, \( C_i, \quad i \in [0, n-1], \) may be calculated from
\[ C_{k-1} = \begin{cases} -r_1(0), & \text{for } k = 1 \\ r_{k-1}(s_{k-1}) - r_k(s_{k-1}), & \text{for } k \in [2, n] \to \mathbb{N}, \text{ for } n \geq 2 \end{cases}. \] (7.176)
If \( s \) belongs to a segment with index higher than 1, \( C_{k-1} \) must be calculated iteratively before the solution of \( r(s) \) can be found. The final global solution may now be calculated from

\[
r(s) = \frac{\alpha(s)}{\beta^3} \left( f_k \beta^2 - f_k \otimes w \otimes w - f_k \otimes (P (f_k \otimes w)) \right) - \frac{1}{E \gamma} \sqrt{(f_k - w s)^T (f_k - w s)}
+ \frac{1}{E \gamma} \left( f_k s - \frac{1}{2} w s^2 \right) + C_{k-1},
\]

(7.177)

where \( \{ s \in [0, L] \rightarrow \mathbb{R}, k \in [1, n] \rightarrow \mathbb{N} | s \in [s_{k-1}, s_k] \} \).

Spatial variation in the distributed load

The solution given above is only valid for constant distributed load along the entire cable. However, an extension to different distributed loads for each segment may be found. Instead of using (7.158), we will assume that the distributed load is constant only within each segment. This means that \( f_0 \) must be calculated from (7.157). Eq. (7.165) must be rewritten to

\[
T \frac{dr}{dp}(s) = f_0 - \sum_{i=1}^{k-1} f_i - \sum_{i=1}^{k-1} w(s_i - s_{i-1}) - w_k(s - s_{k-1}),
\]

(7.178)

where \( \{ s \in [0, L] \rightarrow \mathbb{R}, k \in [1, n] \rightarrow \mathbb{N} | s \in [s_{k-1}, s_k] \} \). We modify (7.163) to

\[
f_k = f_0 - \sum_{i=1}^{k} f_i - \sum_{i=1}^{k-1} w(s_i - s_{i-1}) + w_k s_{k-1}.
\]

(7.179)

Now the derivation goes just like in the previous section, and we end up with

\[
r_k = \int_{s_{k-1}}^{s_k} (f_k - w_k s) \left( \frac{1}{E \gamma} + \frac{1}{\sqrt{(f_k - w_k s)^T (f_k - w_k s)}} \right) ds.
\]

(7.180)

Continuing on the boundary condition and claiming continuity between segments, the integration constants may be calculated as before. The new solution for spatially varying distributed loads with the assumptions made above, are given from

\[
r(s) = \frac{\alpha(s)}{\beta^3} \left( f_k \beta^2 - f_k \otimes w \otimes w - f_k \otimes (P (f_k \otimes w)) \right) - \frac{1}{E \gamma} \sqrt{(f_k - w s)^T (f_k - w s)}
+ \frac{1}{E \gamma} \left( f_k s - \frac{1}{2} w s^2 \right) + C_{k-1},
\]

(7.181)

\[
\beta = \sqrt{w_k^T w_k} = \|w_k\|_2,
\]

(7.182)

\[
\alpha = \ln \left[ \left( \beta s - \frac{1}{\beta} f_k^T w_k \right) + \sqrt{(f_k - w_k s)^T (f_k - w_k s)} \right],
\]

(7.183)

\[
P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}.
\]

(7.184)
7.4.3 Catenaries as Boundary Value Problems (BVP)

To this point we have considered catenaries as a cable with known properties and known loads. The goal has been to achieve a solution for the end point. Let us revert this and try to find the force components when the two end points are known. In this section we will assume that the cable consists of only one segment. The governing differential equations are still known, see (7.168). The problem has now changed from a ODE/IVP to a ODE/BVP (see Section 10.1.1). An analytical solution to this problem could be found if \( f_k \) was solved from (7.168). Nobody has published such analytical solution, and we will have to deal with numerical solutions of the problem. Both MATLAB’s function \texttt{bvp4c} or e.g. a finite element solution will do if the cable’s geometry is of particular interest.

Sometimes other problems arises when we are dealing with boundary value problems. We may for instance know the boundary coordinates, but want to find the end force to yield this solution. The ODE/BVP has now increased to include an estimation of the parameter \( \bar{f}_1 \). A shooting method based on the accurate analytical solution might be applied. We will now given an example where we use MATLAB’s routine to estimate this force.

**Example 7.1 3D catenary BVP with end force estimation.**

A cable of length 10 meters is fastened in \((0,0,0)\). The cable’s weight is \( 1 \text{N/m} \), and \( EA = 7.854 \times 10^4 \). The end force is \( \bar{f}_1 = [10 \ 0 \ 10]^T \text{N} \). Eq. (7.177) gives the end point \((8.8137,0,4.1421)\). MATLAB’s routine \texttt{bvp4c} was applied to these parameters, and an estimate of the end force was found. The results are shown in Figure 7.25. Even if the initial guess for the estimated force deviated from the correct value, the routine managed to find very good estimates. The results were dependent on the numerical option and the mesh in the variable \( s \). The MATLAB code for this solution is shown below.

![Figure 7.25: Solution of the boundary value problem for a hanging cable. MATLAB’s routine \texttt{bvp4c} was applied. The dashed line shows the initial guess for a solution.](image-url)
function cable3dbvp
W=[0 0 -1]'; %Constant distributed force
Fguess=[5 0 8]'; %Guess for end force (We know this guess is wrong)
E=1e9; %Modulus of elasticity
D=0.1; %Cable's diameter
A=pi*(D/2)^2; %Cable's cross-sectional area
L=10; %Cable's length
EndP=[8.8137 0 4.1421]'; %Boundary point for second end.

s=0:.1:L; %Initial mesh

%Calculates an initial guess for the cable's geometry (a straight line)
%Initial mesh

n=length(s);
Xguess=linspace(0,EndP(1),n);
Yguess=linspace(0,EndP(2),n);
Zguess=linspace(0,EndP(3),n);
sol.x=s;
sol.y=[Xguess; Yguess; Zguess];
sol.parameters=Fguess;

%Plots the initial guess
figure(1)
cf
plot(sol.y(1,:),sol.y(3,:),'k--')
xlabel('x')
ylabel('z')
title('Cable bvp')

sol = bvp4c(@cableODE,@cableBC,sol,[],W,E,A,L,EndP); %Solves the BVP

%Plots the result
hold on
plot(sol.y(1,:),sol.y(3,:),'k-')
hold off

disp(sprintf('Calculated end force is ...
[%.2f %.2f %.2f] N \n',sol.parameters(1),sol.parameters(2),sol.parameters(3)))

%System's ODEs
function drds = cableODE(s,r,F,W,E,A,L,EndP)
Fk=F+W*L;
drds = (Fk-W*s)*(1/sqrt((Fk-W*s)'*(Fk-W*s))+1/(E*A));

%System's boundary values

223
function res = cableBC(ya,yb,F,W,E,A,L,EndP)
res = [ya(1)
ya(2)
ya(3)
yb(1)-EndP(1)
yb(2)-EndP(2)
yb(3)-EndP(3)];

7.4.4 Hydrodynamic Drag Loads
Consider the current acting on a small cable element $dp$ in Figure 7.26. Hydrodynamic loading on mooring lines are usually modelled with the so-called cross-flow principle. This principle assumes that the flow separates due to cross-flow past the cable, that the longitudinal current components do not influence on the transverse forces on a cross-section, and that the transverse forces on a cross-section is mainly due to separated flow effects on the pressure distribution around the ship Faltinsen [66]. From Morison’s equation we get

$$df_{dt} = -\frac{1}{2} C_{DT} \rho_w d (dp) |\mathbf{v} \cdot \mathbf{t}| |\mathbf{v} \cdot \mathbf{t}| \mathbf{t} = -\frac{1}{2} C_{DT} \rho_w d (dp) |\mathbf{v}_t| \mathbf{v}_t, \quad (7.185)$$
$$df_{dn} = -\frac{1}{2} C_{DN} \rho_w d (dp) |\mathbf{v} - (\mathbf{v} \cdot \mathbf{t}) \mathbf{t}| (\mathbf{v} - (\mathbf{v} \cdot \mathbf{t}) \mathbf{t}) = -\frac{1}{2} C_{DN} \rho_w d (dp) |\mathbf{v}_n| \mathbf{v}_n, \quad (7.186)$$

where $\rho_w$ is the density of water, $d$ is the cable’s diameter, $C_{DT}$ and $C_{DN}$ are tangential and normal drag coefficients, $\mathbf{t}$ is tangential vector of the cable, $\mathbf{v}, \mathbf{v}_t,$ and $\mathbf{v}_n$ are velocity, normal velocity and tangential velocity on the cable on vectorial form. Notice that it is common to normalize to the projected area $d \cdot (dp)$, in this kind of hydrodynamic forces. Some refer to $f_{dn}$ as the normal friction force, but this is inaccurate. Friction forces are usually normalized to wet surface of the body. For a cable in the normal direction this leads to

$$df_{df} = -\frac{1}{2} C_{DF} \rho d \pi (dp) |\mathbf{v} - (\mathbf{v} \cdot \mathbf{t}) \mathbf{t}| (\mathbf{v} - (\mathbf{v} \cdot \mathbf{t}) \mathbf{t}) = -\frac{1}{2} C_{DF} \rho d \pi (dp) |\mathbf{v}_n| \mathbf{v}_n, \quad (7.187)$$

where $C_{DF} = \frac{1}{4} C_{DN}$.
Hydrodynamic forces may not be handled in the same way as gravity in the catenary equations. Gravity acts on each cable segment independently of it’s orientation. This means that the
shape and tension may be solved explicitly. Calculation of current forces requires information on the cable segment’s orientation, and calculation of orientation requires knowledge of the current forces. Consequently, an iteration procedure is needed.

The calculation of hydrodynamic forces (7.185) and (7.186) assumes that the water’s velocity is constant over the actual segment. For analytical solutions the hydrodynamic forces have to be constant over each segment. This means that the density of segments should be higher in areas with high curvature, and lower in areas with low curvature. For the calculation, it is natural to apply the vector between the segment boundaries as the vector $t$.

![Figure 7.27: Division of the cable into one segment shows that the current will in this case have no effect when the straight line between the nodal points are used as the tangential vector, $t$, in the expression for hydrodynamic forces. Two segments yields a slightly better result.](image)

When the analytical solution is applied to a problem which involves high curvature on the cable, the effect of current forces should be iterated to obtain a good result. First, a geometry should be calculated with no influence from the current. In the next step the current forces should be turned on, and a new geometry will be obtained. This will again make the basis for the next solution, and so on. The iteration can be stopped when the difference between two consecutive solutions is low.

### 7.5 Mooring System for FPSOs

Generally, a mooring system consists of $n_m$ lines connected to the structure and horizontally spread out in a certain pattern. The anchor lines are composed of chain, wire lines or synthetic material, often partitioned into several segments of different types and properties. For turret-moored ships, when the turret is rotatable, the relative angle between the turret and the body-fixed frame is given by the *turret angle* $\alpha_{tu}$. The length of each anchor line is adjusted by winches and determines the pre-tension and thus the stiffness of the mooring system. The anchor lines enter the turret through fairleads below the hull and the coordinates are defined as *terminal points* (TP). Mooring lines are subjected to three types of excitation (Triantafyllou [316]): Large amplitude LF motions, medium amplitude WF motions and small amplitude, very high frequency vortex-induced vibrations. For the purpose of PM control system design it is appropriate to consider the mooring lines’ influence on the low-frequency vessel model.

A horizontal-plane spread mooring model can be formulated as

$$\tau_{moor} = -R^T(\psi)g_{mo}(\eta) - d_{mo}(\nu),$$

(7.188)
where $\tau_{mo} \in \mathbb{R}^3$ is the vector of generalized mooring forces, $d_{mo} \in \mathbb{R}^3$ represents the additional damping in the system due to the mooring system, and $g_{mo} \in \mathbb{R}^3$ is the Earth-fixed restoring term

$$g_{mo} = \sum_{i=1}^{nm} \begin{bmatrix} H_i \cos \beta_i \\ H_i \sin \beta_i \\ H_i \bar{x}_i \sin \beta_i - H_i \bar{y}_i \cos \beta_i \end{bmatrix},$$

(7.189)

which is the vectorial sum of the force contribution from each line. $H_i$ is the horizontal force at the attachment point of the ship along the direction of line $i$, and $\beta_i$ is the earth-fixed direction of the line. $\bar{x}_i$ and $\bar{y}_i$ are the corresponding moment arms. In a quasi-static approach, by disregarding the dynamic effects in the mooring lines, the restoring forces $g_{mo}$ are treated as function of the low-frequency ship position and heading $\eta$ only according to

$$g_{mo}(\eta, \alpha_{tu}),$$

(7.190)

where the horizontal force contributions $H_i$ in (7.189) are replaced by the static line characteristics (distance/force relationships) for each line $i$ by

$$H_i = f_{Hi}(h_i),$$

(7.191)

which is a function of the horizontal distance $h_i$ between TP and the anchor of each line. About a working point the line characteristics (7.191) can be linearized by

$$H_i = \bar{H}_{oi} + \frac{df_{Hi}}{dh_i} \bigg|_{h_i=h_{io}} \Delta h_i,$$

(7.192)

where $\bar{H}_{oi}$ is the average horizontal force in the working point $h_{io}$, and $\frac{df_{Hi}}{dh_i} \bigg|_{h_i=h_{io}}$ is the slope of the line characteristics (7.191) at $h_{io}$. By assuming fixed anchor line length and neglecting the influence of the current field along the line profile, the generalized mooring forces (7.188) in a working point can be approximated by a 1st-order Taylor expansion of the static restoring mooring forces and the mooring damping about the working points $\eta = \eta_o$ and $\nu = 0$ according to

$$\bar{g}_{mo}(\eta) = \bar{g}_{mo}(\eta_o) + \mathbf{G}_{mo}(\eta - \eta_o) + h.o.t.$$

$$d_{mo}(\nu) = \mathbf{D}_{mo} \nu + h.o.t.,$$

(7.193)  
(7.194)

where h.o.t. denotes higher order terms and

$$\mathbf{G}_{mo} = \left[ \frac{\partial \bar{g}_{mo}}{\partial \eta} \right]_{\eta=\eta_o}, \quad \mathbf{D}_{mo} = \left[ \frac{\partial d_{mo}}{\partial \nu} \right]_{\nu=0}.$$  

(7.195)

For simplicity, the Earth-fixed frame is often placed in the natural equilibrium point of the mooring system, i.e. $\bar{g}_{mo}(\eta_o = 0) = 0$. $\mathbf{D}_{mo}$ and $\mathbf{G}_{mo}$ are the linearized mooring damping and stiffness matrices assumed to only contribute in the horizontal-plane. They can, for symmetrical mooring patterns about the $xz$- and $yz$- planes be formulated as

$$\mathbf{G}_{mo} = \text{diag} \{ g_{m11} \ g_{m22} \ 0 \ 0 \ 0 \ g_{m66} \},$$

$$\mathbf{D}_{mo} = \text{diag} \{ d_{m11} \ d_{m22} \ 0 \ 0 \ 0 \ d_{m66} \}. $$

(7.196)  
(7.197)
Hence, the quasi-static mooring model can be written

$$\tau_{\text{moor}} = -R^T(\psi)G_{\text{mo}}\eta - D_{\text{mo}}\nu.$$  \hspace{1cm} (7.198)

For further details in modelling, see Faltinsen [66], Huse and Matsumoto [122] and Triantafyllou and Yue [317]. For fully dynamically positioned vessels with no anchor system, $\tau_{\text{moor}}$ is equal to zero.