

# Chapter 2

## PRINCIPLES OF RADARS

**Keywords.** Radar, Radar equation, Antenna gain, Radar cross-section

### 2.1 Introduction

The word RADAR is an acronym for Radio Detection (A)nd Ranging. A Radar is an electromagnetic system for the detection and location of objects. It operates by transmitting a particular type of waveform and detects the nature of the echo signal. Radars can operate in situations like darkness, fog, rain, or when the object is located far away. In such situations the human eye is almost useless. However, perhaps the most important attribute of a radar is that it can also measure the distance or range to an object.

A radar consists of three main parts:

- A transmitting antenna.
- A receiving antenna
- An energy detecting device, or a receiver.

The transmitting antenna emits electromagnetic radiation, a portion of which is reflected back by the target. The receiving antenna receives this reflected energy and delivers it to the receiver. The receiver processes this energy to detect the presence of the target and to extract its location, relative velocity, and other information. The energy emitted by the radar is usually in the form of a train of narrow, rectangular-shaped pulses. This is called a radar waveform(see Fig.2.1) of course, there could be other kinds of radar waveforms too.

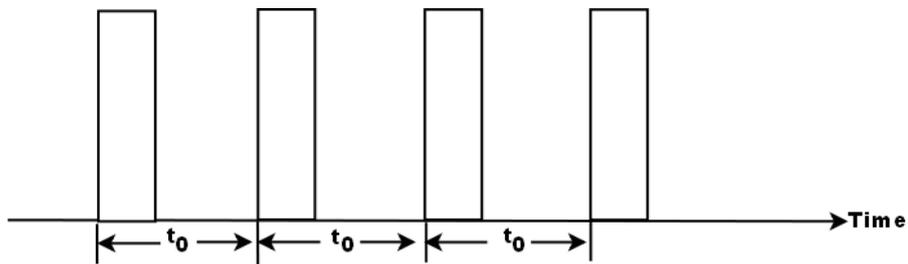


Figure 2.1: A typical radar waveform

This figure should be understood to mean that a pulse of electromagnetic energy is being transmitted every  $t_0$  seconds. It also means that the frequency of transmission (that is, the number of pulses per second) is given by

$$f_p = \frac{1}{t_0} \quad (2.1)$$

For example, if a pulse is sent every 0.1 seconds then it also means 10 pulses are being sent every one second, and so  $f_p=1/0.1=10$ . Consider a pulse of energy being sent at a given instant in time. It travels to the target at a speed of  $c$  meters/sec, hits the target, and is reflected back at the same speed. The reflected energy is received at the radar  $T_R$  seconds after sending the pulse(see Fig.2.2).

Then , the distance or range  $R$  to the target is given by

$$R = \frac{cT_R}{2} \quad (2.2)$$

where,  $R$  is in meters and  $c$  is usually taken to be the speed of light and is assumed to be  $c = 3 \times 10^8 m/sec$  In deriving the above equation we assumed that only one pulse was transmitted and was later received after reflection from the object. But usually a number of pulses are sent at regular intervals, as was shown in Fig. 2.1. In Fig. 2.3(a), the first pulse, after being reflected from the target, is received by the radar before the second pulse is transmitted. There will be no ambiguity here as the reflected pulse can be easily identified as a reflection of the first pulse. But in Fig.2.3 (b), we notice that the reflection of the first pulse is received after the second pulse has been transmitted. This causes some confusion since the radar, without any additional information, cannot determine whether the received signal is a reflection of the first pulse or of the second pulse. This leads to an ambiguity in determining the range. Note that this ambiguity does not arise if  $T_R < t_0$ ,

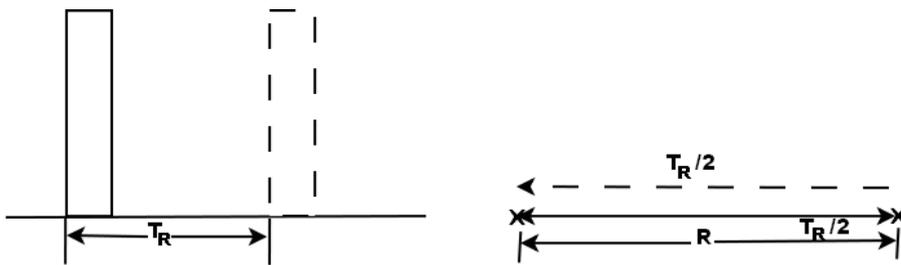
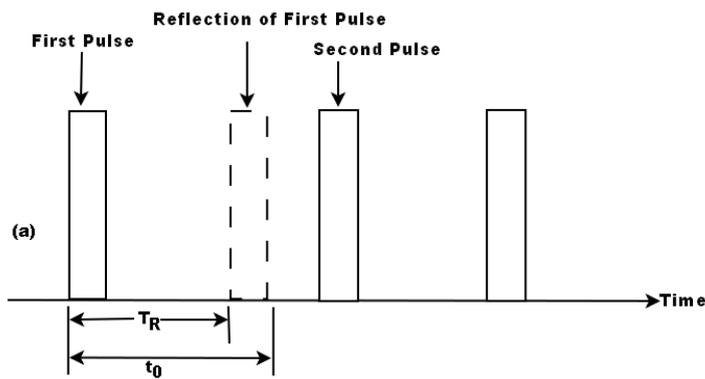


Figure 2.2: Transmitted and received pulses

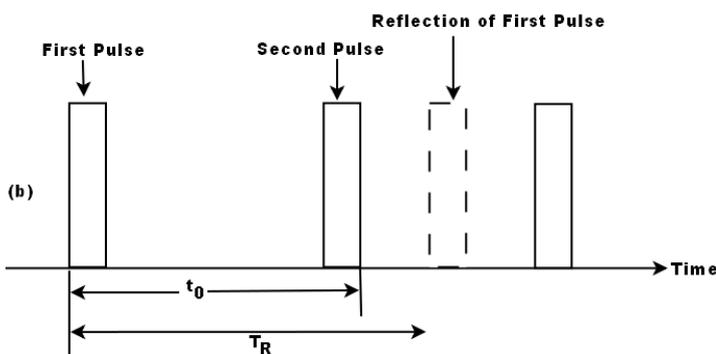
in which case the reflection of the first pulse is always received before the second pulse is transmitted. Thus, the maximum range or distance of the target which does not cause any ambiguity is denoted by  $R_{unamb}$  and is given by,

$$R_{unamb} = \frac{ct_0}{2} = \frac{c}{2f_p} \quad (2.3)$$

This is known as the maximum unambiguous range of the radar. If the



(a) No ambiguity in range measurement



(b) ambiguity in range measurement

Figure 2.3: (a) No ambiguity (b) ambiguity in range measurement

target is beyond this distance then the reflection of a pulse is received after the next pulse has been transmitted. This is known as the second-time-around echoes effect.

EXAMPLE 2.1 : Consider a radar with pulse repetition frequency 1000 Hz. (a) Find the time duration between two pulses. (b) Suppose an echo from a distant object is received 20  $\mu$  sec after a pulse is transmitted, what is the distance of the object from the radar? (c) Is there a second -time-around echo from this object?

ANSWERS The pulse repetition frequency  $f_p = 1000$  Hz. (a) The time duration between pulses is given by

$$t_0 = \frac{1}{f_p} = \frac{1}{1000} = 0.001\text{sec} = 1\text{msec} \quad (2.4)$$

(b) The echo is received after  $T_R = 20\mu\text{sec} = 20 \times 10^{-6}$  sec. Remember that  $T_R$  is the time taken by the pulse to cover the distance from the radar to the object and back. Hence, the time taken by the pulse to travel one way (i.e., from the radar to the object) is half of  $T_R$ . Since the speed of propagation is  $c = 3 \times 10^8$  m/sec, the distance of the object from the radar is given by,

$$R = \frac{cT_R}{2} = \frac{3 \times 10^8 \times 20 \times 10^{-6}}{2} = 3000\text{m} = 3\text{km} \quad (2.5)$$

(c) A second-time-around echo occurs only when the distance of the object is more than the maximum unambiguous range of the radar. Also remember that the  $R_{unamb}$  is that distance of an object for which the echo comes back exactly  $t_0$  seconds after being transmitted. Hence,

$$R_{unamb} = \frac{ct_0}{2} = \frac{0.001 \times 3 \times 10^8}{2} = 150 \times 10^3 \text{ m} = 150 \text{ km} \quad (2.6)$$

Since the distance of the object is much less than  $R_{unamb}$ , there is no second-time-around echo.

COMMENTS: The example given above is simple and yields a solution quickly. However, a few comments are in order.

(1) However simple a formula might be, do not just write it down from memory, substitute the appropriate values, and get the answer. This way no doubt you will get the correct answer, but it will not help to improve your understanding of the physics behind the formula. A better way is to derive the formula yourself through logical reasoning, before applying it. This way

you need not memorize formulas at all. You can derive them yourself in a few moments.

(2) Be careful about the units you use. Regardless of the units used in the problem statement, first convert the given values to some standard system of units (e.g., msec to sec, km to m, etc.) and then substitute them in the formula. Always mention the units for any numerical value you write.

(3) Remember and apply the above two comments while solving the exercises given in these lecture notes!

## 2.2 The Radar Equation

The radar equation relates the range of a radar to the characteristics of the transmitter, receiver, antenna, target, and the environment in which the radar operates. The radar equation is useful

- in determining the distance of the target from the radar.
- as a tool for understanding radar operation.
- in serving as a basis for radar design.

Consider a radar using a transmitting antenna which radiates power uniformly in all directions. Such antennas are called isotropic antennas. Let  $P_t$  be the power radiated by such an antenna. Then the power density at a distance  $R$  is given by,

$$\hat{P}_d = \frac{P_t}{4\pi R^2} \quad (2.7)$$

This is apparent from Fig.2.4 given below.

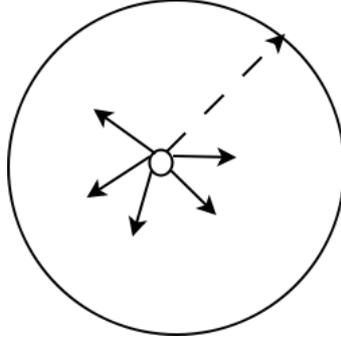


Figure 2.4: Power density at distance R from the radar

Note that at a distance R, the power  $P_t$  is uniformly distributed over an area given by the surface area of a sphere of radius R. Hence, we get the equation or  $\hat{P}_d$  as above.

However, it is somewhat wasteful to radiate energy in all directions. Thus, radars may employ directive antennas to channelize, or direct, the radiated power in a particular direction (i.e., the direction of the target). The gain in power density so achieved is denoted by G and is a measure of the increased power radiated in the direction of the target as compared to the power that would have been radiated from an isotropic antenna. It may also be defined as the ratio of the maximum radiation intensity from the given antenna to the Radiation intensity from a lossless isotropic antenna with the same power input. Here, radiation intensity is defined as the power radiated per unit solid angle in a given direction. The factor G is also known as the antenna gain. Thus, the power density from a directive antenna at a distance R is given by

$$P_d = \hat{P}_d G = \frac{P_t G}{4\pi R^2} \quad (2.8)$$

The target, situated at a distance R, intercepts a portion of the power and reflects it in various directions. The measure of the amount of power

intercepted by the target is defined as the radar cross-section of the target. It is denoted by  $\sigma$  and has the unit of area. Note that the radar cross-section is the characteristic of a particular target and is a measure of its size as seen by the radar. Thus, the amount of power intercepted by the target at a distance  $R$  from the radar is,

$$\hat{P} = P_d \sigma = \frac{P_t G \sigma}{4\pi R^2} \quad (2.9)$$

A simple way to understand this equation is to assume that the target has a surface area  $\sigma$  on which radiations of density  $P_d$  impinge. However, like all "simple" explanations, this statement is not precise in the sense that  $\sigma$  is not just the surface area that the target presents to the radar radiations, but  $\sigma$  is a complex function of the target surface area as well as many other factors which depend on the characteristics of the target.

Now we assume that this power  $\hat{p}$  gets radiated in all directions, and therefore, using the same argument, the power density of the reflected signal at the receiving antenna is given by

$$P_d^r = \frac{\hat{P}}{4\pi R^2} = \frac{P_t G \sigma}{(4\pi R^2)^2} \quad (2.10)$$

The radar antenna now captures a portion of this reflected power. How much of this power is actually captured depends on what is known as the effective area of the receiving antenna. This is denoted by  $A_e$  and has the unit of area. It is also known as the antenna effective aperture. The power  $P_r$  received by the radar is,

$$P_r = P_d^r A_e = \frac{P_t G \sigma A_e}{(4\pi R^2)^2} = \frac{P_t G \sigma A_e}{(4\pi)^2 R^4} \quad (2.11)$$

The radar receiver must be capable of detecting the power received. Suppose the radar receiver can detect only those signals which are greater than a value  $S_{min}$  (known as the minimum detectable signal), then the maximum range of the radar can be obtained from

$$S_{min} = \frac{P_t G A_e \sigma}{(4\pi)^2 R_{max}^4} \quad (2.12)$$

From which,

$$R_{max} = \left[ \frac{P_t G A_e \sigma}{(4\pi)^2 S_{min}} \right]^{\frac{1}{4}} \quad (2.13)$$

This is the fundamental form of the radar equation. Note that the two important antenna parameters used here are the antenna gain  $G$  and the effective antenna aperture  $A_e$ .

Many radars use the same antenna for both transmission and reception. In such cases, from antenna theory, the relationship between the antenna gain and the receiving effective area of an antenna is given as,

$$G = \frac{4\pi A_e}{\lambda^2} \quad (2.14)$$

where,  $\lambda$  is the wavelength of the transmitted energy. Substituting this relation in (2.13), we obtain another form of the radar equation.

$$R_{max} = \left[ \frac{P_t A_e^2 \sigma}{4\pi \lambda^2 S_{min}} \right]^{\frac{1}{4}} \quad (2.15)$$

If we substitute for  $A_e$  instead of  $G$  then from (2.14), we get

$$A_e = \frac{G \lambda^2}{4\pi} \quad (2.16)$$

and obtain the radar equation as,

$$R_{max} = \left[ \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right]^{\frac{1}{4}} \quad (2.17)$$

The above radar equation must be interpreted somewhat carefully. Note that in (2.15)  $R_{max}$  appears to be inversely proportional to  $\sqrt{\lambda}$  whereas in (2.17) it appears to be directly proportional to  $\sqrt{\lambda}$ , and in (2.13) it is independent of  $\lambda$ . The reason behind this apparent anomaly is the following: when we speak of the variation of a particular parameter with respect to another, we assume that all other parameters are constants. This is possible only when these parameters are independent. In this case it is not so, since the dependence between  $G$ ,  $A_e$ , and  $\lambda$  is governed by (2.14) and any variation in one of these parameters has to affect at least one of the other two. Hence, we cannot speak of the variation of  $R_{max}$  with respect to  $\lambda$  only (or with respect to  $G$  or  $A_e$  only.)

#### EXAMPLE 2.2

(a) Find the power density at a target situated at a distance of 50km from a radar radiating a power of 100 MW from a lossless isotropic antenna.

(b) If this radar now employs a lossless isotropic antenna with a gain of 5000 and the target has a radar cross-section of 1.2 m<sup>2</sup>, then what is the power density of the echo signal at the receiver?

(c) If the minimum detectable signal of the radar is  $10^{-8}$  MW and the wavelength of the transmitted energy is 0.02 m, then what is the maximum range at which the radar can detect targets of the kind mentioned in (b)?

(d) What is the effective area of the receiving antenna?

(e) Suppose, due to some modifications made in the radar system components, the antenna gain is doubled while keeping the antenna effective

aperture constant. Find the new radar range.

(f) What is the new radar range if the antenna gain doubles while  $\lambda$  remains constant?

ANSWER

(a) Power radiated by the radar  $P_t = 100 \text{ MW} = 100 \times 10^6 \text{ W}$ .

Distance of the target =  $R = 50 \text{ Km} = 50 \times 10^3 \text{ m}$ .

Power density at the target

$$= \hat{P}_d = \frac{P_t}{4\pi R^2} = \frac{100 \times 10^6}{4\pi \times (50 \times 10^3)^2} \quad (2.18)$$

$$= 0.3183 \times 10^{-2} \text{ W/m}^2 \quad (2.19)$$

(b) Antenna gain  $G = 5000$ .

Radar cross-section of the target =  $\sigma = 1.2 \text{ m}^2$

Power density at the target when a directive antenna is used

$$= P_d = \hat{P}_d G = 0.3183 \times 10^{-2} \times 5000 = 15.915 \text{ W/m}^2. \quad (2.20)$$

The amount of power intercepted by the target

$$= \hat{P} = P_d \sigma = 15.915 \times 1.2 = 19.098 \text{ W}. \quad (2.21)$$

This power is now reflected back to the receiving antenna. Hence, the power density of the echo signal at the receiver

$$= P_d^r = \frac{\hat{P}}{4\pi R^2} = \frac{19.098}{4\pi \times (50 \times 10^3)^2} = 6.079 \times 10^{-10} W/m^2. \quad (2.22)$$

(c) The wavelength of the transmitted energy,  $\lambda = 0.02m$ .

The minimum detectable signal  $S_{min} = 10^{-8}mW = 10^{-11}w$ .

Then the maximum range  $R_{max}$  is given by (2.17) as

$$R_{max} = \left[ \frac{P_t G^2 \lambda^2 \sigma}{(4\pi)^3 S_{min}} \right]^{1/4} = \left[ \frac{100 \times 10^6 \times (5000)^2 \times (0.02)^2 \times 1.2}{(4\pi)^3 \times 10^{-11}} \right]^{1/4} \quad (2.23)$$

$$= 88183.6m = 88.1836Km. \quad (2.24)$$

(d)From (2.13), the effective area of the receiving antenna

$$= A_e = \frac{G\lambda^2}{4\pi} = \frac{5000 \times (0.02)^2}{4\pi} = 0.0159m^2. \quad (2.25)$$

(e) Let the new antenna gain be  $G' = 2G$ , and the corresponding wavelength be  $\lambda'$ .The new radar range  $R'_{max}$  can be found by using either (2.10),). If we use (2.13) then we get,

$$\frac{R'_{max}}{R_{max}} = \left[ \frac{G'}{G} \right]^{1/4} = 2^{1/4} = 1.1892. \quad (2.26)$$

So,

$$R'_{max} = 1.1892 \times R_{max} = 1.1892 \times 88.1836 = 104.8Km. \quad (2.27)$$

If we use (2.15) then we get,

$$\frac{R'_{max}}{R_{max}} = \left[ \frac{\lambda^2}{\lambda'^2} \right]^{1/4}$$

Since  $A_e$  is constant, from (2.14) we get

$$\frac{G'}{G} = \frac{\lambda^2}{\lambda'^2} = 2$$

Which, on substitution, leads to the same result. Similarly, from (2.17) we get

$$\frac{R'_{max}}{R_{max}} = \left[ \frac{G'^2 \lambda'^2}{G^2 \lambda^2} \right]^{1/4} = \left[ 2^2 \cdot \frac{1}{2} \right]^{1/4} \quad (2.28)$$

which, again, leads to the same result.

(f) Here, we will use (2.17) to obtain,

$$\frac{R'_{max}}{R_{max}} = \left[ \frac{G'^2}{G^2} \right]^{1/4} = (2^2)^{1/4} = 1.414 \quad (2.29)$$

Hence,

$$R'_{max} = 1.414 \times R_{max} = 1.414 \times 88.1836 = 124.7Km \quad (2.30)$$

Note that exactly the same result can be obtained by using either (2.13) or (2.15).

It should be understood that the actual radar range is much smaller than the  $R_{max}$  obtained from the radar equation. The reason is that the above simplified equations do not take into account a number of important factors which reduce the range of operation. These will be dealt with later.